EUROPEAN ORGANIZATION FOR NUCLEAR RESEARCH
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CERN - ST Division

AIRFLOW IN THE CNGS TUNNEL STRUCTURES
- A STUDY FOR ENSURED OPERATIONAL SAFETY -

R. Blom

Abstract

In the CNGS project, the nature of neutrinos is to be studied. These neutrinos are created by a radioactive decay, and to control the path of the particles that decay into neutrinos, a large under-pressure tunnel is needed. To have such a large evacuated volume, where the pressure is kept at 100 Pa, connected with areas accessible for the personnel, might be hazardous. The purpose of this study is to quantify, by means of CFD simulations, the airflow, air velocity, pressure and temperature that would arise in the tunnels in case of a rupture in the sealing of the evacuated tunnel. For the simulated case, where the titanium window that seals the tunnel disappears completely in an accident, the pressure drops to 0.987 bar in the area accessible to the personnel after 7 seconds. An airflow velocity of 50.9 meters per second accompanies this pressure drop. As possible means of personnel protection, the effects of using safety baffles have been studied. These measures would, with a 50 percent close off, lead to a reduction of the sudden pressure drop by 38 percent and a reduction of the maximum velocity by 15 percent.

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1   INTRODUCTION

The CNGS project is a project carried out in cooperation between CERN and the Italian institute for particle and nuclear physics research (INFN) in Gran Sasso. The aim of the CNGS project is to study neutrino oscillation. CERN's part in the project is to supply the neutrino beam needed for this purpose.

To minimize the loss of pions and kaons through interactions with air when creating the neutrino beam, an evacuated decay tunnel is envisaged for the CNGS project. This tunnel contains a steel pipe similar to a large water main or oil pipe. The pipe, usually referred to as the "decay tube", will be 992 m long and have a diameter of 2.45 m and it will be sealed into the rock. At the exit of the decay tube, a thick steel flange will close it off. The entrance to the decay tube will be sealed off by a 1.45 m diameter titanium window, with a thickness of 2 mm. This thin window is mounted on a flange which, in turn, is welded to the decay tube. Via the target chamber, the decay tube is connected to other underground structures and the SPS accelerator environment in which personnel are frequently present. The closest such structure is the ECA4 cavern, about 900 metres upstream of the decay tube entrance window.

Having such a large evacuated volume, where the pressure is only 100 Pa, connected to areas accessible to people imposes great security risks that have to be evaluated. To do so, it has been carried out a CFD simulation of the air flow that will arise in case of a rupture of the titanium window that seals of the decay tube from the ambient tunnels at atmospheric pressure.

1.1   Problem settings

In the case of a window rupture, the flow in the first moments can be compared to that of a shock tube, with the low pressure side being the decay tube and the high pressure side being the other underground structures

The evolution of the flow in the tunnel up until the time when the compression and the rarefaction waves start to interact can be described in the following three main steps:

1.   When the window brake at t = 0 seconds, a compressible wave is formed in the vacuum tunnel. This wave propagates into the low-pressure volume. At the same time a rarefaction wave is formed which propagates into the high-pressure volume in figure 1.1.

2.   At t = tr the compressible wave reflects against the end wall of the vacuum tube and progresses back towards the window.

3.   At some time, t > 2tr the reflected wave reaches the former high-pressure part of the tunnel. And some time later, the reflected compression wave will start to interact with the rarefaction wave and eventually an equilibrium pressure will be reached.

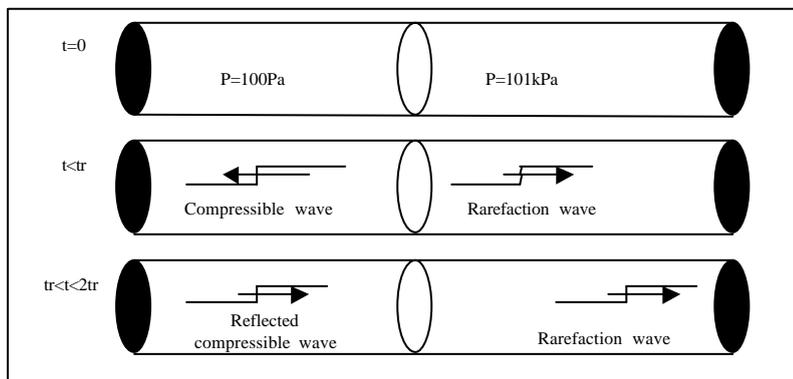A description of the flow before and between these three steps described above is given in figure 1.1 below.

Figure 1.1 The schematics of the wave propagation in the tube. The time tr is the time for reflection of the compressible wave at the end of the decay tube.

1.2 Aim of the project

This project is carried out to predict the effect of the airflow at the ECA4 cavern in the case of a rupture of the titanium window. The ECA4 cavern is connected to the underground structures of CNGS, where people will be present under the CNGS operation. The study will be carried out with simplified geometry and similar flow conditions, chosen conservatively so that the effects from the airflow are not underestimated. The final aim of the study is to obtain parameters describing the flow, such as pressure and temperature variations, as well as velocities and accelerations arising. These parameters will be used to determine whether the risk of any danger to the personnel can be ruled out, or the actions are needed to ensure the safety of the personnel.

1.3 Assumptions

The study is subject to a number of assumptions.

Firstly, the analysis will only include the decay tube and the main structures that connect it with the ECA4 cavern. Any effects from and on other structures will be ignored. Furthermore, the geometry studied will be simplified though the effective cross sectional areas will be maintained.

Secondly, the analysis will be restricted to cover only the case of transient, turbulent and compressible flow, with constant viscosity. No gravitational effects will be included.

Finally, no attempt will be made to determine the health effects from the flow on the personnel in the area or on the tunnel structures. This is subject of request to the safety division (TIS) at CERN.

2 THEORETICAL ANALYSIS

In this chapter the framework and the method used for the theoretical analysis are described. These are followed by a presentation of results and conclusions.

2.1 Introduction

For the theoretical analysis the problem settings are simplified and carried out as an analysis of a closed shock tube. The shock tube analysis is based on the case of a straight tunnel containing two volumes separated by a membrane (i.e. the titanium window). Specifically, the study is concerned with the case when the media have significantly different pressures and densities, so that shock and rarefaction waves will develop and progress in the tube in the case of a rupture of the membrane (titanium window). This situation is similar to that just after a rupture of the titanium window in the CNGS tunnel structure. The results can give an insight to the conditions that can be expected in the tunnels in the first moments after a rupture.

2.2 Input to analysis

Flow medium	air
Pressure at high pressure side, P_4	= 101000 Pa
Pressure at low pressure side, P_1	= 100 Pa
Temperature at both sides, T_4 and T_1	= 293 K
Density at high pressure side ρ_4	= 1.247 kg/m^3 Ref. [1]
Ratio of specific heats, γ	= 1.4 Ref. [1]
Sound speed at high pressure side	= 340.6 m/s Ref. [1]

2.3 Further assumptions

For the theoretical analysis the following simplifications have been made.
- The flow is considered inviscid.
- The air is considered to be an ideal gas.
- Isentropic conditions are assumed.
- The structure is considered to be closed, i.e. no inlet exists, and to have a constant
- diameter.
- The membrane separating the two volumes is considered to cover the entire diameter
- of the tube.
- No gravitational effects are included.

2.4 Description of analysis

The main computational steps for determining the parameters characterizing the flow are described below. A more thorough description of the analysis is given in appendix 1.
1.   Density and sound speed are calculated in the low-pressure volume.
2.   The Mach number of the shock wave is calculated
3.   Based on the Mach number, the pressure after the shock, the velocities of the shock, the front of the rarefaction wave and the airflow in the region behind the shock are calculated. Furthermore, temperatures and densities are determined in the different regions.

This analysis gives sufficient information for a theoretical understanding of the flow up until the point of reflection.

2.5 Results

The calculated Mach number for the flow is approximately six. Figure 2.1 presents an overview of the different regions identified in the analysis together with a wave diagram describing the progress of the flow in the tube as a function of time. The wave diagram is only valid up to t=1.31 seconds when the shock wave is reflected at the end of the tube, but the figure has been drawn for a longer time for clarity.

Further information about the flow in the different flow regions obtained from the theoretical analysis is presented in table 2.1. The regions referred to in the table are the regions presented in figure 2.1 below.
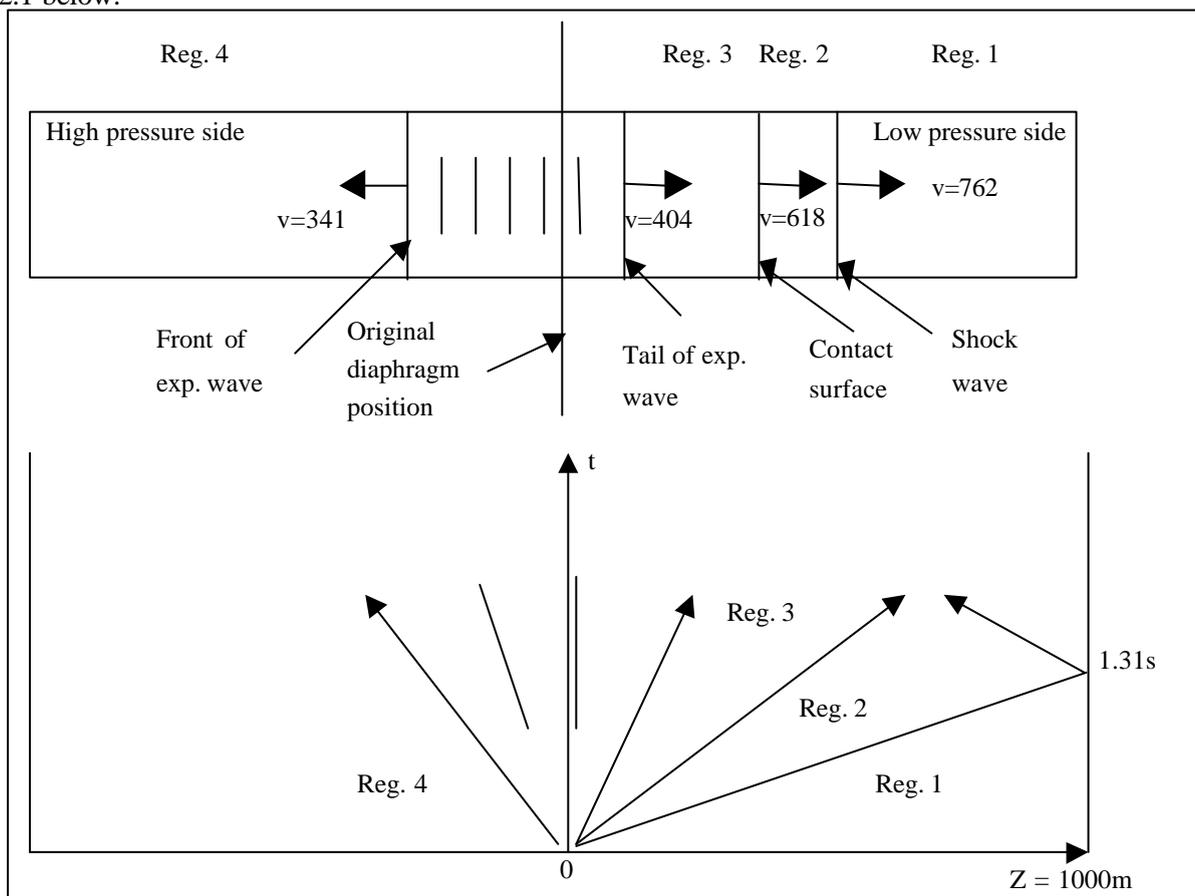

Figure 2.1 Description of regions dividing the flow and a wave diagram describing the progresses of the various entities as a function of time. All velocities are given in m/s.

Table 2.1 Flow parameters from theoretical analysis.

	Reg. 1	Shock wave	Cont. surface	Rarefact. tail	Reg. 3	Rarefact. front	Reg. 4
Pressure [Pa]	100	-	4299	-	4299	-	101000
Velocity [m/s]	0	762	618	404	618	-340.6	0
Sound speed [m/s]	125	-	-	-	215	-	340.6
Density [kg/m^3]	0.009	-	-	-	0,131	-	1.25
Temperature [K]	293	-	119	-	119	-	293

2.6 Conclusions

From the theoretical analysis important conclusions can be drawn regarding the character of the flow the values of pressures, velocities and temperatures during the first stages of the flow that would arise should the titanium window break.

According to the calculated Mach number (6.082), shock waves will definitely be present in the flow. The shock wave will travel into the low-pressure volume, followed by the tail of the rarefaction wave, which travels at approximately half the velocity of the shock wave. Furthermore, the pressure will rise abruptly by a factor of approximately 43 during the shock. This implies that temperatures and velocities will be unsustainable in the area near the decay tube window.

The front of the rarefaction wave will travel towards the control room with the speed of sound. The analysis holds no exact information on the pressure or temperature and its gradients over the rarefaction wave. However, the fact that gradients in pressure and temperature will be present is obvious as the total change over the rarefaction wave is more than 95 kPa in pressure and 174 K in temperature. The analysis also implies that high airflow velocities are to be expected in ECA4.

3   CFD SIMULATIONS

In this chapter, a description of the simulations of the airflow in the CNGS tunnels is given. The first subchapter gives information about assumptions, approximations and settings for the simulation model. An evaluation of the simulations follows this. Finally, the results from the simulations are presented and discussed and conclusions are drawn based on the results from the simulation.

3.1   Simulation model

There are a number of parameters needed to describe a simulation model. The mesh describes the geometrical shape and divides the volume into cells for the calculation. A characterization of the flow situation together with the boundary and initial conditions sets the physical constraints and starting points for the simulation. Finally, the numerical solution's controls prescribe the computational approach to the problem. These aspects in the simulation model have important effects on the results and are presented below.

3.1.1   Geometrical model and mesh

The geometrical model that all the simulations are based upon only includes the main structures connecting the decay tube with the ECA4. Specifically, these are; the decay tube, the target chamber, the junction chamber, the access gallery, the proton beam tunnel (TT41, TT40 and TZ40), the ECA4 and the PAM4 that connect the ECA4 with the surface (see figure 3.1 and [5] for details). To model these structures, simple geometrical shapes have been used, and care has been taken to maintain the correct physical dimensions and flow areas of the structures.
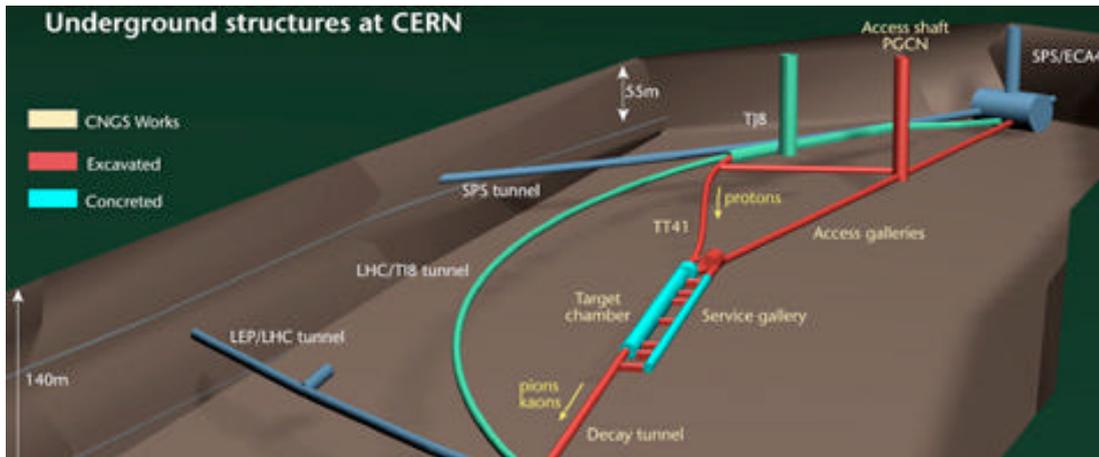
Figure 3.1 Underground tunnel structures

Apart from the accuracy in physical dimensions, there are a number of considerations that need to be taken into account when creating a mesh for simulations. Firstly, the meshing has to be implemented in such a way that it optimizes the numerical computations of the flow field. This includes for example the general choice of cell distribution for different regions as well as considerations of side length and volume ratios for the cells. Moreover, the mesh has to be tuned to fulfil the demands for the modelling of physical aspects such as for example the turbulence modelling close to the walls. These criteria must be satisfied and, at the same time, the number of cells has to be minimized to keep short the computational time.

A so-called "butterfly" shape (see figure 3.1) has been chosen for the cross sections of all structures but the ECA4. This mesh is easy to create and it is known to show high computational stability as it gives a good side length ratio for the cells and it avoids the sharp tips in the cells at the centre that would otherwise be formed. Furthermore, the rectangular section in the centre has been distorted to provide better side length and volume ratios. The outermost cells have been made thinner to allow the code to capture the turbulence correctly. The cells have been made as long as possible without causing divergence in the calculations and where it has shown to be necessary, lengthwise incremental refinements have been used.
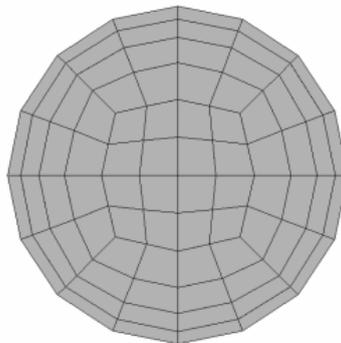
Figure 3.2 Typical butterfly mesh in cross section.

The area of the cross-sections has been matched to the areas of the actual tunnels, with an estimated reduction in area to compensate for the space occupied by magnets and other equipment. The reduction in percent, as well as the effective radius and area after the reduction for the altered structures are presented in table 3.1 below. No changes have been made to the structures not mentioned in the table.

Table 3.1 Changes on cross-sections

	Area	Red. of area	Effective radius
Target chamber	28.72 m^2	30 %	2.505 m
Junction chamber	45.48 m^2	20 %	3.4 m
Proton beam tunnel	5.325m^2	28%	1.1025m

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For the ECA4, an automatically generated tetrahedral mesh has been used, as this volume is less structured and it is harder to predict the directions of flow in it.

Some of the simulations include the insertion of wall segments in the mesh. They have been simulated by impermeable cell baffles, which satisfy the no-slip condition [2].

3.1.2 Boundary and initial conditions

The boundary conditions implemented in the model are of two types, wall and pressure boundaries. The wall boundaries form the outer surfaces of the model, and they satisfy the no-slip condition. The pressure boundary is imposed at the top of PAM4 to simulate an open atmosphere.

The initial conditions describe the temperature and the pressure in the volumes on both sides of the titanium window before the time of rupture, i.e. at the start of the simulation. The temperature is set to 293K on both sides while the pressure at the high-pressure side is set to 101000 Pa and the pressure on the low-pressure side is set to 100 Pa. This transition from 100Pa to atmospheric pressure is done stepwise over four cells, from 5 kPa to 61 kPa, in order to be able to obtain convergence in the first time steps. The values for the two intermediate cells are based on the theoretical analysis as well as on simulations, and correspond to the values expected in these regions at the rupture of the entrance window.

3.1.3 Media properties and numerical settings

The medium used for the computations is air. The air is treated as an ideal gas with values depending on both temperature and pressure. The mol weight is set to 28.96kg/kmol. The flow characteristics implemented in the simulation are transient, turbulent, viscid and compressible. The viscosity is considered constant.

In the choice of schemes for the calculations the main priority has been robustness due to the difficulties with the strong pressure gradient at the start of the simulation. PISO, the Pressure Implicit Split Operator, is the only alternative for simulations of transient flows in StarCD. For the inner iterations however a number of choices of differencing schemes are available. In the simulations carried out, the mars scheme [2] has been used for the momentums as well as the temperature and density. Details on the settings for these schemes are given in appendix 2.

To achieve a balance between reaching convergence in the calculations and maintaining a reasonable physical time step, the time step has been varied for different phases of the development of the flow. The simulations were started with a time step of 0.00001 seconds, and then the time step was gradually increased to 0.0005 seconds.

3.2 Validity aspects

The computational validity of the simulation can be determined by a study of a number of control variables. For the simulations performed, the most crucial of these are the Courant number, the number of iterations needed for convergence in each time step and the number of iterations needed to reach convergence between each time step. In table 3.2 below, the maximum values for the four most critical control variables are given for each model configuration (as described in chapter 3.3, 3.4 and 3.5), together with the allowed number of iterations for these.

Table 3.2 Number of iterations for validity parameters

	NSP	NST	PISO	COURANT NB
Full open	9	6	5	1.53
Reduced open	48	5	4	11.7
Baffle 50 %	64	8	7	7.49
Baffle 60 %	64	7	6	7.18
Allowed	10000	10000	5000	300

Convergence in the solvers is set to be achieved when the residual is smaller than 10E-5 when solving for velocities and 10E-12 for all other solvers. Further information on the computational settings can be found in appendix 2.

3.3   Results for open flow model

The results presented in the paragraphs below are for the open flow model. This model is extremely pessimistic. It assumes that the full 2.45 metre diameter of the decay tube will become immediately and fully open at the moment of rupture (i.e. both the window and the supporting flange will disappear). No obstacles are present in the tunnels.

Results from two different sections, the ECA4 and the entrance to the access gallery (i.e. at a location very near ECA4) are presented. The simulations has been run for a physical time of 90 seconds, but results are only shown in this report for the first 20 seconds as these cover the extreme values.

3.3.1   Results at ECA4

The pressure distribution for a transversal slice in ECA4 and the minimum and maximum pressure variations in ECA4 are shown in figure 3.3 below. The red area is unaffected, while the other colours show an under-pressured area relative to atmospheric pressure. As it can be seen, only a small region of the ECA4, is affected by the rarefaction wave. The lowest pressure reached in this region is 0.95 bar, which is reached approximately 5.3 seconds after the rupture. The pressure drop from atmospheric pressure to 0.95 bar takes place over 2.7 seconds.
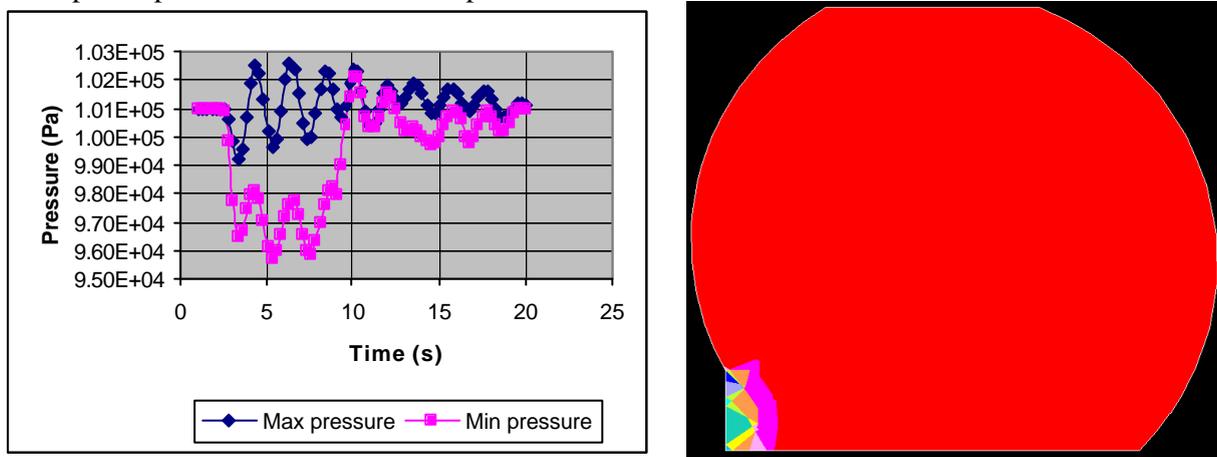

Figure 3.3 Pressure distribution at t=5.3s and pressure variation over time in ECA4

In figure 3.4, the variation of the minimum and maximum temperatures and absolute velocities are shown. During the entire sequence the temperature in ECA4 is between 16 and 24 Celsius, and the maximum velocity is 79.5m/s.
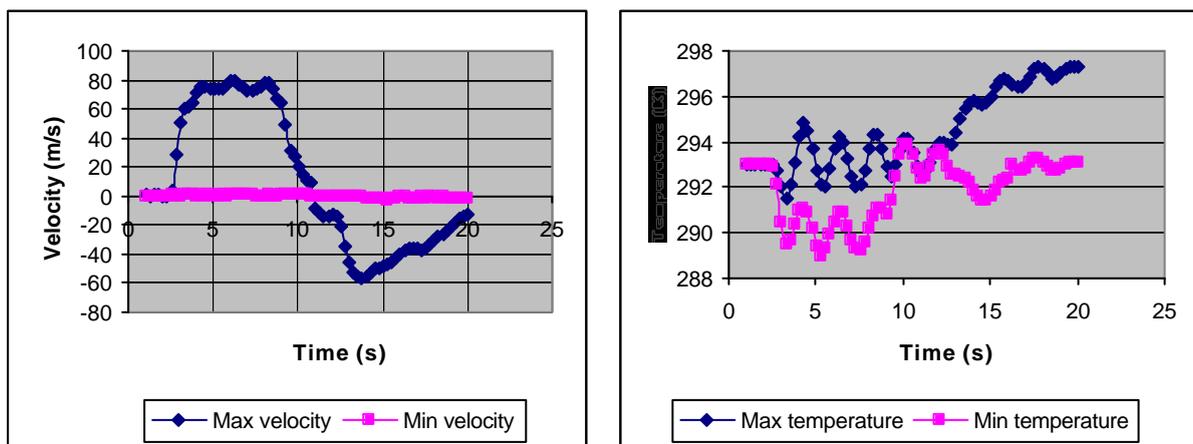
Figure 3.4 Velocity and temperature variation over time in ECA4

3.3.2 Results in tunnel entrances

Figure 3.5 below shows the minimum and maximum pressure and velocity variations in a region of the access gallery near ECA4. The values are more extreme than those in ECA4.
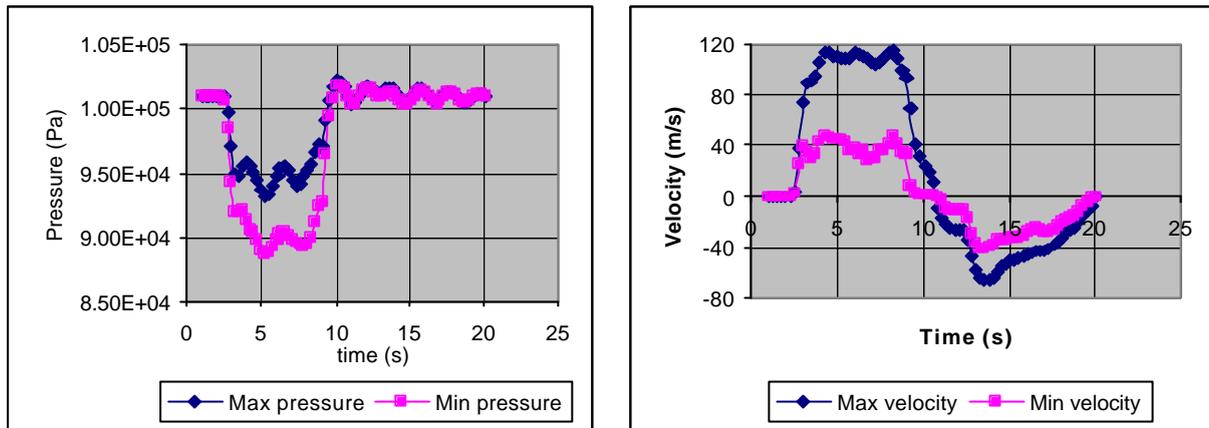
Figure 3.5 Pressure and velocity variation over time in the tunnel entrances

The minimum pressure is here 0.89 bar occurring at the same time as the minimum for ECA4. The maximum velocities are 115 m/s into the tunnel (pos. sign) and 62 m/s out of the tunnel (neg. sign). The temperature varies between 12 and 26 Celsius.

3.4 Results for open flow model with baffles

The following results are based on two simulations where baffles have been implemented in the model. This should approximately simulate protective walls, which might be installed in the access gallery and the proton beam tunnel, but which can only cover a certain fraction of the tunnels' cross sections for construction reasons. The two options investigated are a 65 % and a 50 % coverage. The baffle in the proton beam tunnel is situated as far away from the titanium window as possible, however still remaining inside the CNGS tunnels. The baffle in the access gallery is situated approximately 110 meters from ECA4. Apart from the introduction of these baffles, the model is the same as the open flow model, and the results are directly comparable.

Results on pressure drop and air speed (velocity) in the same two locations as in chapter 3.3 are presented, i.e. in the ECA4 and at the access gallery entrance

3.4.1 Results at ECA4

The results for minimum pressure and maximum velocity variation in ECA4 with 65 and 50 percent baffles are shown in figure 3.6.
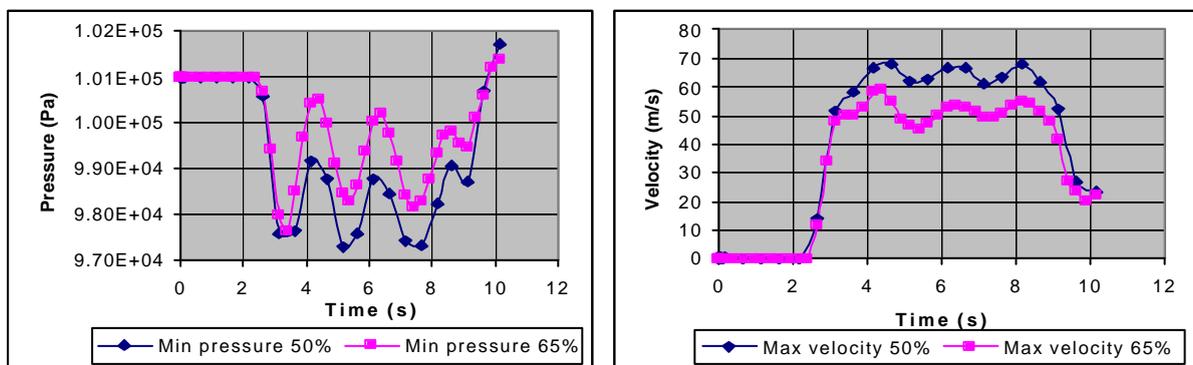
Figure 3.6 Pressure and velocity variations over time in ECA4 for simulations with two baffle configurations

The minimum pressure and maximum velocity with the 65 percent configuration are 0.977 bar and 59.4 m/s respectively. With the 50% configuration these values reach 0.973 bar after 5.14 seconds and 67.6 m/s. The temperature is within the interval of 18 to 21°C with 65% baffles and between 17 and 21 Celsius with 50% baffles.

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The results for minimum pressure and maximum velocity variation at the entrance to the access gallery with 65 and 50 percent baffles are shown in figure 3.7. The lowest pressure and velocity with the 65% configuration is here 0.949 bar and 92.7 m/s. The 50% configuration reaches 0.924 bar and 107 m/s.

The temperature is within the interval of 15 to 21 Celsius with 65% baffles and between 13°C and 21°C with 50% baffles.
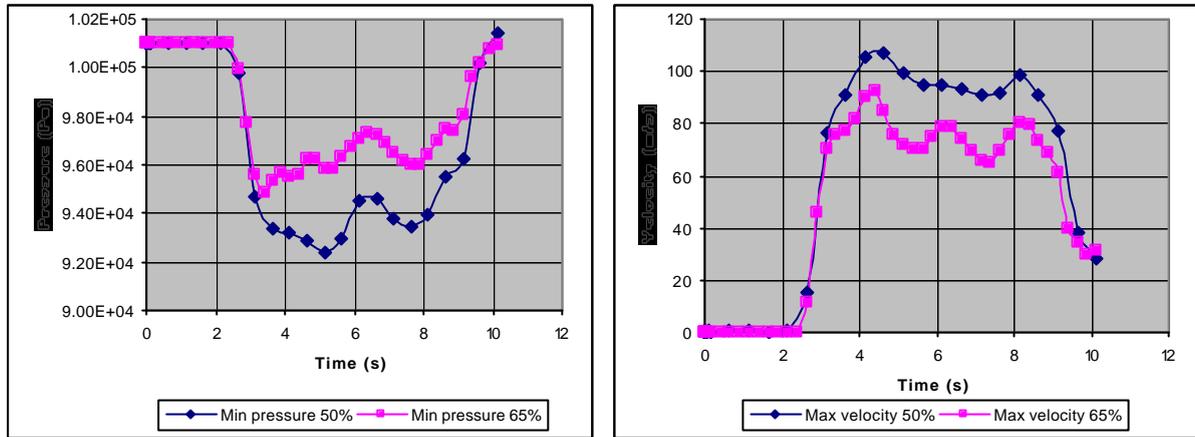
Figure 3.7 Pressure and velocity variations over time at the entrance to the access gallery with baffles in the proton beam tunnel and the access gallery.

3.5   Results for reduced opening model

The results presented in the following subchapters are from simulations where the open diameter of the entrance window at rupture is only 1.496 metres. This is a more realistic case than in section 3.3, since the diameter approximately corresponds to the actual diameter of the titanium window (the small difference is due to the already existing mesh, from the simulations presented in 3.3). Apart from this, the model is directly comparable to the open flow model that produced the results presented in chapter 3.3. The reduction of the opening in the simulations has been implemented by use of cell baffles, which for modelling reasons has been placed 0.195 meter into the decay tube. This is also the new position of the pressure gradient.

Results are presented for the same two locations as in chapter 3.3 and 3.4.

3.5.1 Results at ECA4

The minimum pressure and maximum velocity variation in ECA4 is shown in figure 3.8.
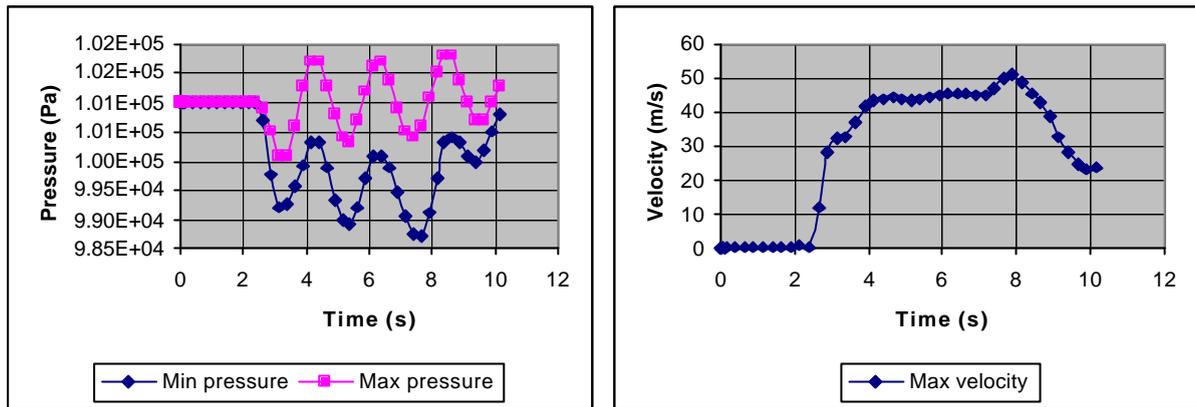
Figure 3.8 Pressure and velocity variations in ECA4 for the simulation with a reduced opening.

The lowest pressure reaches 0.987 bar, and the highest occurring velocity is 50.9 m/s. The temperature is within 18 to 21 Celsius in ECA4.

3.5.2 Results at the access gallery entrance

Figure 3.9 below shows the minimum pressure and the maximum velocity variations at the tunnel entrance. The minimum pressure is 0.964 bar and the maximum velocity is 71 m/s. The temperature is between 17 and 20 Celsius.
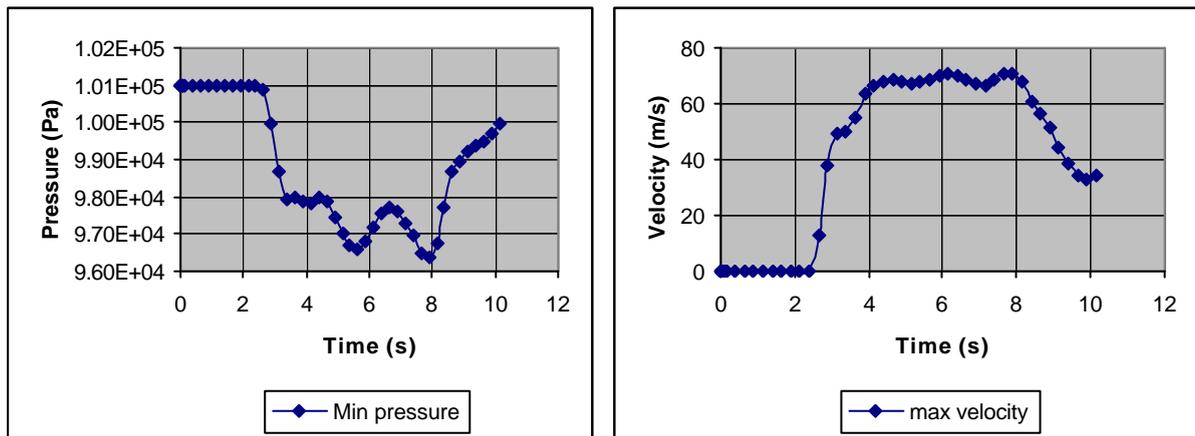
Figure 3.9 Pressure and velocity variation at the access gallery entrance for the reduced opening simulation.

3.6 Summary of results

A summary of the pressures (minima), velocities (maxima) and temperatures (ranges) in the ECA4 cavern for all simulation models can be found in table 3.3 below. The equivalent values for the access gallery entrance are presented in table 3.4.

Table 3.3 Summary of results at ECA4

Eca4	Full open	Reduced open	Baffle 50 %	Baffle 65 %
P	0.95 Bar	0.987 Bar	0.973 Bar	0.977 Bar
At t =	5.3 s	7.64 s	5.14 s	3.4 s
V	79.5 m/s	50.9 m/s	67.6 m/s	59.4 m/s
At t =	5.3 s	7.9 s	8.19 s	4.4 s
T	16-24 °C	18-21 °C	17-21 °C	18-21 °C

Table 3.4 Summary of results at the access gallery entrance

Tunnel	Full open	Reduced open	Baffle 50 %	Baffle 65 %
P	0.89 Pa	0.964 Pa	0.924 Pa	0.949 Pa
At t =	5.3 s	7.89 s	4.64 s	3.4 s
V	115 m/s	71 m/s	107 m/s	92.7 m/s
At t =	5.3 s	7.89 s	4.64 s	4.4 s
T	12-26 °C	17-20 °C	13-21 °C	15-21 °C

3.7   Conclusions

As for the numerical validity of the simulations, none of the equations exceed the maximum allowed numbers of iterations in any simulation. This implies that convergence has been reached in each time step and that the numerical solutions are reliable.

It is clear that the conditions in ECA4 and at the tunnel entrance are affected both by the open diameter assumed at the moment of rupture of the window, and by the introduction of baffles (protection walls) in the tunnels.

The pressure drop in the ECA4 with 1.496 meters opening diameter is almost a third of the one with a 2.45 meters opening.

The introduction of baffles that cover 50 percent of the tunnel sections gives a drop in pressure of about 38 percent compared to that with the full opening (2.45 m) and no baffles. If 65 percent of the tunnels are covered, the reduction in pressure drop is slightly higher. The velocities are reduced by 15 percent for the 50 percent baffle coverage and by about 25 percent for the 65 percent baffles. The introduction of baffles will give a locally higher flow speed, which gives a rarefaction that travels quicker toward the ECA4, and therefore the extreme values occur earlier.

If the same baffle alternatives were implemented in the simulation with the reduced opening (1.496 m), one can expect roughly the same changes in pressure and velocity as between full open and full open with baffles models. In the ECA4, this would then give a pressure drop by only approximately 0.016 bar, the lowest pressure reaching 0.996 bar, with the introduction of baffles covering 50 percent of the tunnels. The value for the maximum velocity would be approximately 43m/s for this case.

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APPENDIX 1 THEORETICAL CALCULATIONS

INPUT

Flow medium	air
Pressure at high pressure side, P_4	= 10100Pa
Pressure at low pressure side, P_1	= 100Pa
Temperature, T4 and T1	= 293K
Density at high pressure side, rho4	= 1.247kg/m^3
Ratio of specific heats, gamma	= 1.4
Sound speed at high pressure side	= 340.6m/s

ASSUMPTIONS AND SIMPLIFICATIONS

The flow is considered inviscid
The air is considered as an ideal gas
Isentropic conditions are assumed
The tube is considered as closed, i.e. no inlet exists.

LIST OF VARIABLES

Gamma	Ratio of specific heats
c	Sound speed
Rho, ρ	Density
P_1	Pressure in
T_1	Temperature
M	Mach number
1,2,3,4	Region for which the variable is valid

CALCULATION OF SOUND SPEED IN LOW PRESSURE REGION 1

The density in zone 1 is calculated using formula 1.1 below formed using the equation of state, $P=\rho RT$ [1] and the assumed condition of an isentropic process

$$\text{rho1} := \text{rho4} \cdot \left(\frac{P1}{P4} \right)^{\frac{1}{\text{gamma}}} \tag{1}$$

rho1 = $8.911 \cdot 10^{-3}$ kg/m^3.

The speed of sound, c, in a compressible medium is given by $c^2 = (\partial P/\partial \rho)_s$ [1], where the subscript s indicates that the derivative is taken at constant entropy. By combining this equation with the equation of state given above the speed of sound in the low-pressure region can be calculated [1]

$$c1 := \left(\text{gamma} \cdot \frac{P1}{\text{rho1}} \right)^{0.5} \tag{2}$$

which gives c1 = 125.344 m/s.

MACH NUMBER

The Mach number for the flow into the low pressure area can be calculated based on the ratio of the pressure in the high and low pressure sides [3]

$$\frac{P4}{P1} = \frac{\left[2 \cdot \text{gamma} \cdot M^2 - (\text{gamma} - 1) \right]}{\text{gamma} + 1} \cdot \left[1 - \frac{(\text{gamma} - 1)}{(\text{gamma} + 1)} \cdot \frac{c1}{c4} \cdot \left(M - \frac{1}{M} \right) \right]^{-2 \cdot \frac{\text{gamma}}{(\text{gamma} - 1)}} \tag{3}$$

which gives M = 6.082.

PRESSURES

Using the Mach number, the pressure in the region after the shock can be determined according to the formula given below [1]

$$P2 := \frac{P1 \cdot \left[2 \cdot gamma \cdot M^2 - (gamma - 1) \right]}{gamma + 1}$$

(4)

which gives P2 = 4,299 kPa. This pressure is equal to the pressure in region 3 so, P3 = 4,299 kPa.

VELOCITIES

The velocity of the shock is given by the product of the Mach number and the sound speed in the medium in front of the shock, us = M·c1. This gives a propagation speed of us = 762.341 m/s for the shock.

The velocity of the medium in region 2, ie. after the shock has passed, is given by (Thompson, p. 424)

$$u2 := \frac{2}{gamma + 1} \cdot c1 \cdot \left(M - \frac{1}{M} \right)$$

(A.4)

which gives u2 = 618,11 m/s.

The velocities of the contact surface and of the air behind the contact surface are the same as that in region 2 so, ucs = u3 = 618,11 m/s.

The head of the expansion wave moves in the opposite direction of the shock with the speed of sound relative to the flow speed of the medium in region 4. This gives uef = c4 = 340.6 m/s as the medium in region 4 is at rest.

The tail of the expansion wave moves in the opposite direction of the shock with the speed of sound in region 3 relative to the flow speed of the medium in region 3. This medium moves in the direction of the shock with a speed of u3 = 618.11 m/s. The speed of sound in region 3 is calculated using formulas (1) and (2).

$$rho3 := rho4 \cdot \left(\frac{P3}{P4} \right)^{\frac{1}{gamma}}$$

(5)

which gives rho3 = 0.131 kg/m^3. When the density is known, the sound speed can be calculated

$$c3 := \left(gamma \cdot \frac{P3}{rho3} \right)^{0.5}$$

(A.6)

which gives c3 = 214.506 m/s.

Finally, the speed of the tail of the expansion wave is determined as uet = c3-u3 = -253.51 m/s. The minus sign implies that the tail of the expansion wave moves in the direction of the shock.

TEMPERATURES
The temperature behind the expansion wave, ie. in region 3, can be obtained considering the expansion as isentropic and making use of the equation of state

$$T3 := T4 \cdot \left(\frac{P3}{P4} \right)^{\frac{(gamma-1)}{gamma}}$$

<div align="right">(A.7)</div>

which gives T3 = 118.895 K

An approximation of the temperature in region 2 directly after the shock can be found using values for the change of temperature from [1], Appendix D. Using values for M=4 gives T2 = 293·4.047 = 1185.8.

The actual temperature under the given assumptions that is to be expected in the medium in region 2 after the passing of the shock should be higher than this value due to the method and mach value used.

SUMMARY OF RESULTS

The Mach number for the shock is 6.082. A summary of other results is given in table 1. The regions referred to in table 1 are presented in figure 1 together with a wave diagram describing the progress of the flow in the tube as a function of time. The wave diagram is only valid up to t=1.13 seconds when the shock wave is reflected at the end of the tube, however, it has been drawn for a longer time for clarity.

Table A.1 Summary of results.

	Reg. 1	Shock wave	Reg. 2	Raref. tail	Reg. 3	Raref. front	Reg. 4
Pressure [Pa]	100	-	4299	-	4299	-	101000
Velocity [m/s]	0	762	618	404	618	-340.6	0
Sound speed [m/s]	125	-	-	-	215	-	340.6
Density [kg/m^3]	0.0089	-	-	-	0,131	-	1.25
Temperature [K]	293	-	>1186	-	148	-	293
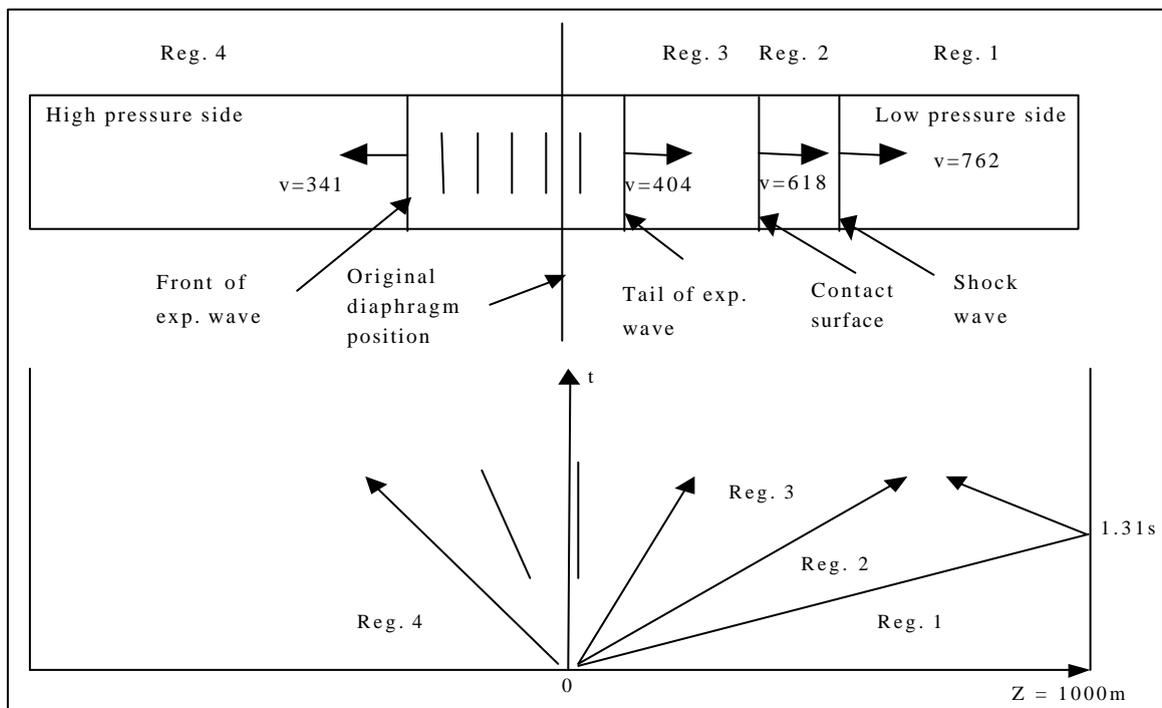
Figure 1 Description of regions dividing the flow and a wave diagram describing the progresses of the various entities as a function of time.

APPENDIX 2 STAR RUN-FILE

```
 *** YOU ARE RUNNING 64 BIT PRECISION VERSION ***
 *** ROUND-OFF LEVEL IN CGST/CGSTV SOLVERS CHANGED FROM DEFAULT
9.999999960041972E-013  TO   1.000000000000000E-005

 *** GEOMETRICAL CALCULATIONS STARTED
 *** GEOMETRICAL CALCULATIONS COMPLETED


|-----------------------------------------------------------------------------
| ------------------------- PROBLEM SPECIFICATION SUMMARY -------------
|-----------------------------------------------------------------------------
|  CASE TITLE .................. =>     ansmodlvis
| | NUMBER OF CELLS ............. =>   165398
|
|  NUMBER OF BOUNDARY FACES .... =>    29767
|
|  MESH DIMENSIONS                  XMIN    XMAX    YMIN    YMAX
ZMIN     ZMAX    |
|       (IN METRES) ............ => -2.8E+02  3.4E+00 -3.4E+00  7.3E+01
0.0E+00  1.9E+03  |
|  RESTART DATA ................ =>    WILL BE SAVED ON FILE.pst
|
|  SURFACE DATA ................ =>    WILL NOT BE SAVED
|
|  BOUNDARY DATA ............... =>    WILL NOT BE PRINTED
|
|  CONVERGENCE DATA ............ =>    WILL NOT BE PRINTED
|
|  FIELD DATA .................. =>    WILL BE PRINTED
|
|  TRANSIENT FLOW (G.RATE)...... =>    START FROM TIME STEP =         0
|
|  DATA DUMP (FILE.pst) ........ =>    EVERY  100 TIME STEPS
|
|  SOLUTION PROCEDURE .......... =>    PISO  (MAXCOR =***, URFPCOR =
8.000E-01)
|  MAX. NO. OF TIME STEPS ...... =>      2
|
|  STARTING LOAD STEP NUMBER ... =>      1
|
|   NUMBER OF TIME STEPS ...... =>      2
|
|   TIME STEP SIZE ........... =>    DT = 1.000000E-05 s
|
|   RAMPING .................. =>    OFF
|
|   POST DATA FREQUENCY ....... =>      1
|
|     CELL DATA SELECTED....... => U,  V,  W,  P,  T,DEN,  H,
|
|     SURFACE DATA SELECTED.... =>    YPLUS,
|
|   PRINT DATA FREQUENCY ...... =>      1
|
|     SURFACE DATA SELECTED.... =>    YPLUS,
|
|  NO. OF FLUID MATERIALS ...... =>      1
|
```

```
|-------------------------------------------------------------------------|
|-> FLUID  1              -------------------------------------------------|
|-------------------------------------------------------------------------|
| SOLVE ...................... =>   U,  V,  W,  P, TE, ED,  T,VIS,DEN,     |
|                                                                         |
|    (STATIC ENTHALPY, THERMAL FORM TRANSPORTED)                          |
|                                                                         |
| FLUID FLOW ................. =>   TURB COMPRESSIBLE HIGH RE K-EPS MODEL  |
|                                                                         |
| CHARACTERISTIC LENGTH ....... =>     1.000E+00 m                        |
|                                                                         |
| MONITORING LOCATION ........ =>   103651                                |
|                                                                         |
| REFERENCE PRESSURE .......... =>   PREF = 5.000E+04 Pa                  |
|                                                                         |
| REFERENCE TEMPERATURE ....... =>   TREF = 2.930E+02 K                   |
|                                                                         |
| MOLECULAR VISCOSITY ......... =>   CONSTANT -   MU = 1.810E-05 Pas       |
|                                                                         |
| DENSITY .................... =>   IDEAL GAS: MOLW = 2.896E+01            |
|                                                                         |
| SPECIFIC HEAT .............. =>   CONSTANT -    C = 1.006E+03 J/kgK      |
|                                                                         |
| CONDUCTIVITY ............... =>   CONSTANT -    K = 2.637E-02 W/mK       |
|                                                                         |
| TURBULENT PRANDTL NUMBER..... =>   PRTUR = 9.000E-01                    |
|                                                                         |
|-------------------------------------------------------------------------|
|-> ADDITIONAL FEATURES USED  ---------------------------------------------|
|-------------------------------------------------------------------------|
|   RESIDUAL DUMP FREQUENCY .... =>      100                              |
|-------------------------------------------------------------------------|
|-> USER FORTRAN CODING USED  ---------------------------------------------|
|-------------------------------------------------------------------------|
|   INITIALISED DEPENDENT VARIABLES                                       |
|-------------------------------------------------------------------------|
|-> BOUNDARY TYPES USED  --------------------------------------------------|
|-------------------------------------------------------------------------|
|  WALL,  PRESSURE,                                                       |
|                                                                         |
|  Turbulence intensity and Mixing length specified at pressure boundaries|
|                                                                         |
|  Temperature specified at pressure boundaries                           |
|                                                                         |
|-------------------------------------------------------------------------|
|-> SOLUTION PARAMETERS  --------------------------------------------------|
|-------------------------------------------------------------------------|
| VARIABLE   |    U        V        W        P       TE       ED          |
|-------------------------------------------------------------------------|
| RELA. FAC. |    -        -        -        -        -                   |
|                                                                         |
| DIFF. SCH. |   MARS     MARS     MARS      -        UD       UD         |
|                                                                         |
| DSCH. FAC. | 5.000E-01 5.000E-01 5.000E-01   -     0.00E+00 0.00E+00    |
|                                                                         |
| SOLV. TOL. | 1.000E-01 1.000E-01 1.000E-01 5.000E-02 1.00E-01 1.00E-01  |
|                                                                         |
| SWEEP LIM. |   1000     1000     1000     10000      100      100       |
```

VARIABLE	T	DENS	TVIS	MVIS	CP	COND
RELA. FAC.	-	1.000E+00	1.000E+00	1.000E+00	1.000E+00	1.000E+00
DIFF. SCH.	MARS	MARS	-	-	-	-
DSCH. FAC.	5.000E-01	-	-	-	-	-
SOLV. TOL.	1.000E-01	-	-	-	-	-
SWEEP LIM.	10000	-	-	-	-	-