Parameterization of Secondary Particle Yields & Neutrino Beam Simulation

- Parameterization of secondary particles production on light nuclei
  - Fit of 400 & 450 GeV *p-Be* experimental data
  - Comparison with previous parameterizations
  - Extrapolation to different energy and material
- Neutrino beam simulation with enhanced statistics through phase-space weighting
  - Comparison with existing data and full MC simulations

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Experimental yields of π^-/π^+, K^-/K^+ by 400-450 GeV/c protons on Beryllium

- H.W. Atherton et al., *Precise measurements of particle production by 400 GeV GeV/c protons on Beryllium targets*, CERN 80-07, 1980
  - p = 60, 120, 200, 300 GeV/c
  - p_t = 0, 500 MeV/c
  - target plate length = 40, 100, 300, 500 mm
- G. Ambrosini et al. (SPY), *Measurement of charged particle production from 450 GeV/c protons on beryllium*, Eur. Phys. J. C10 (1999) 605
  - p = 7, 10, 15, 20, 30, 40, *67.5, 135* GeV/c (at p_t = 0)
  - p_t = 0, ±75, ±150, ±225, ±337, ±450, +600 MeV/c (for p = ±15, ±40 GeV)
  - target plate length = 100, (200, 300 mm for thickness dependence study)
  - WANF "T9-like" target (3 needle-shaped bars 10 cm long and 3 mm thick)

The existing parameterization of secondary particle yield

- Simple formulas fitting the yield of charged secondary particles from 400 GeV/c protons on Be targets:
  - Data above 60 GeV/c ($x \approx 0.15$) at p_t = 0 and 500 MeV/c
  - H.W. Atherton et al., CERN 80-07, 1980: thin target approximation

$$\frac{d^2N_{Be}}{dp\ d\Omega} = Ap_{max}(Be^{-Bx})(\frac{2C}{2\pi}x^2e^{-Cp_t^2})$$

  - A.J. Malensek, FN-341,1981: thick target approximation (500 mm)

$$\frac{d^2N_{Be}}{dp\ d\Omega} = Bx\frac{(1-x)^A}{(1+p_t^2/M^2)^8}(1+5e^{-Dx})$$

- *Both fail at low x* due to lack of data

A new parameterization of secondary particle yield from thick targets

- Goal: improvement with respect to previous models at low x (and $p_T \to 0$).

  H.W. Atherton et al., CERN 80-07, 1980, A.J. Malensek, FN-341,1981
- Secondary yield from fit of:
  - π^+ and K^+ invariant cross-section data derived from *Atherton et al.* & *SPY Collaboration* (Beryllium target),
  - π^-/π^+ and K^-/K^+ data
  - K^0_L evaluation from simple parton model.
- Evaluation of tertiary particles production:
  - Comparison of secondary production from targets of different thickness (100, 200, 300 mm and T9) in SPY.
- Extrapolation to other target material and incident proton energy:
  - Known invariant cross-section dependence on Atomic Number and x.
  - Comparison with other available data at different proton energy.

The invariant cross section for secondary production of π^+ and K^+

- Empirical formula based on general physical arguments.

$$\left(E\frac{d^3\sigma}{dp^3}\right)_{Be} = A\frac{(1-x)^\alpha}{(\Lambda+x)^\beta}\left(1+\frac{a}{x^b}p_t+\frac{a^2}{2x^c}p_t^2\right)e^{-\left(\frac{a}{x^b}p_t\right)}$$

  - Approximate factorization in x and p_t
  - $(1-x)^\alpha$ behavior in the forward direction for $x \to 1$
  - $x^{-\beta}$ behavior in for $x \to 0$
  - Exponential fall in p_t for soft interaction including
    - Polynomial interpolation of low p_t behavior
    - Weak correlation with x
- *Useful to compare data with different incident proton energy*

The invariant cross section for secondary production of π^- and K^- and K^0_L

- Empirical formula describing π^+/π^- and K^+/K^- ratios:

$$R(\pi) = 1 + \frac{4}{1 + e^{-\gamma(x-\delta)}} \qquad R(K) = \frac{1}{\gamma(1-x)^{\delta}}$$

  - function of x only
  - $R(\pi)$ and $R(K)$ shapes supported by observations of Ochs (*Nucl. Phys. B118 (1977) 397*).
    - *$R(\pi) \to 1$ for $x \to 0$;*
    - *$R(\pi) \to 5$ for $x \to 1$;*
    - *$R(K) =$ redefinition of A ($= A \cdot \gamma$) and α ($= \alpha + \delta$).*
- K^0_L production evaluated from simple parton model

$$N(K_L^0) = \frac{N(K^+) + (2n-1)N(K-)}{2n} \approx \frac{1}{4}(N(K^+) + 3N(K^-))$$

  - $n = u/d \approx 2$ assuming isospin symmetry (*H. Wachsmuth* in: *N. Doble et al., NA31 int. note 83 (1990)*)

The fit of the experimental data (Atherton et al. & SPY)

- To compare data with different proton energy:
  - from experimental secondary yield to invariant cross section

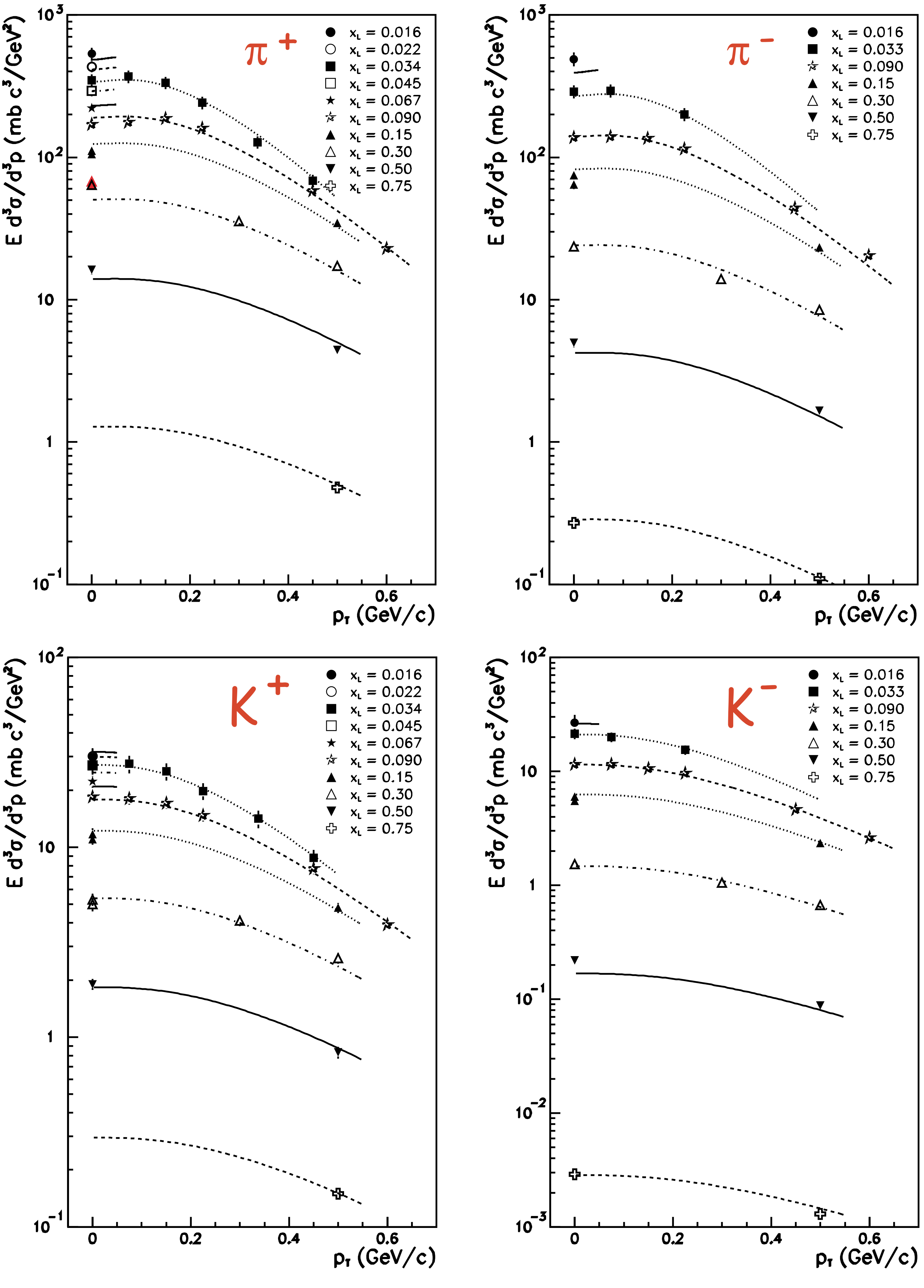
$$\frac{d^2N_{Be}}{dp/p\,d\Omega} = \left(E\frac{d^3\sigma}{dp^3}\right)_{Be}\frac{p^3}{E}\frac{N_0\rho\lambda_p f(L)}{A} \qquad f(L) = \frac{e^{-L/\lambda_s} - e^{-L/\lambda_p}}{1-\lambda_p/\lambda_s}$$

  - *f(L)* = naïve absorption model (100 mm target)
  - tertiary production taken into account (extrapolation to zero thickness from thicker targets)
- Fit parameters:

	A	α	β	Λ	a	b	c	γ	δ
π	91.02	3.216	0.427	0.	5.802	0.175	0.485	6.097	0.469
K	5.551	2.188	0.820	0.99	4.802	0.140	2b	0.870	3.301

- Fit accuracy:
  - $\chi^2/NDF \approx 1$ (experimental error on single measurement $\leq$ 10 %)

Invariant cross-section: fit to data of Atherton et al. & SPY exp. (beryllium targets)



Extrapolations of the parameterization

- Extension to other target material:
  - Known dependence of the invariant cross section on atomic number *A*:

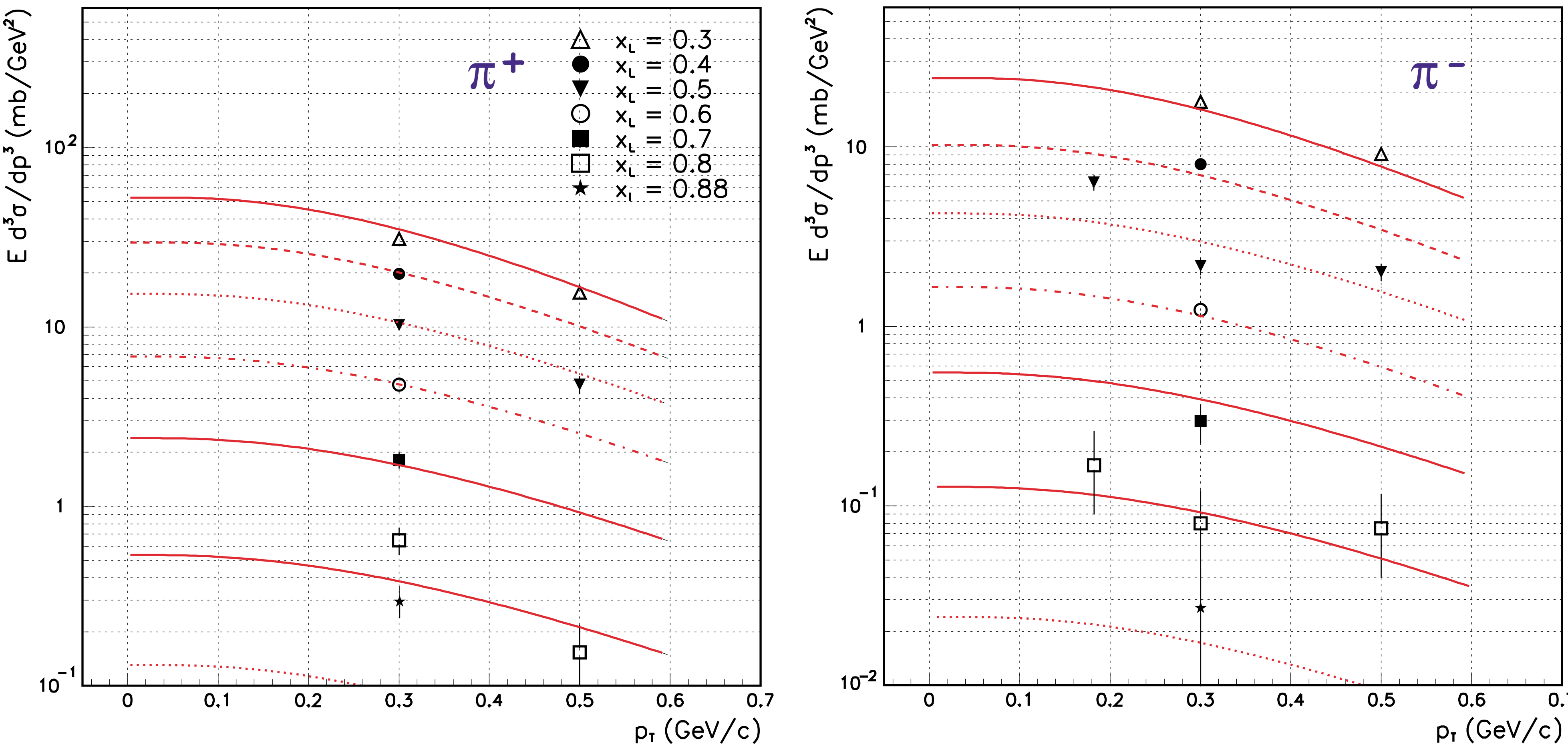
$$\left(E\frac{d^3\sigma}{dp^3}\right)_{A_1} = \left(\frac{A_1}{A_2}\right)^{\alpha}\left(E\frac{d^3\sigma}{dp^3}\right)_{A_2}$$

  - From data on several nuclei: $\alpha(x) \approx 0.74 - 0.55\,x + 0.26\,x^2$

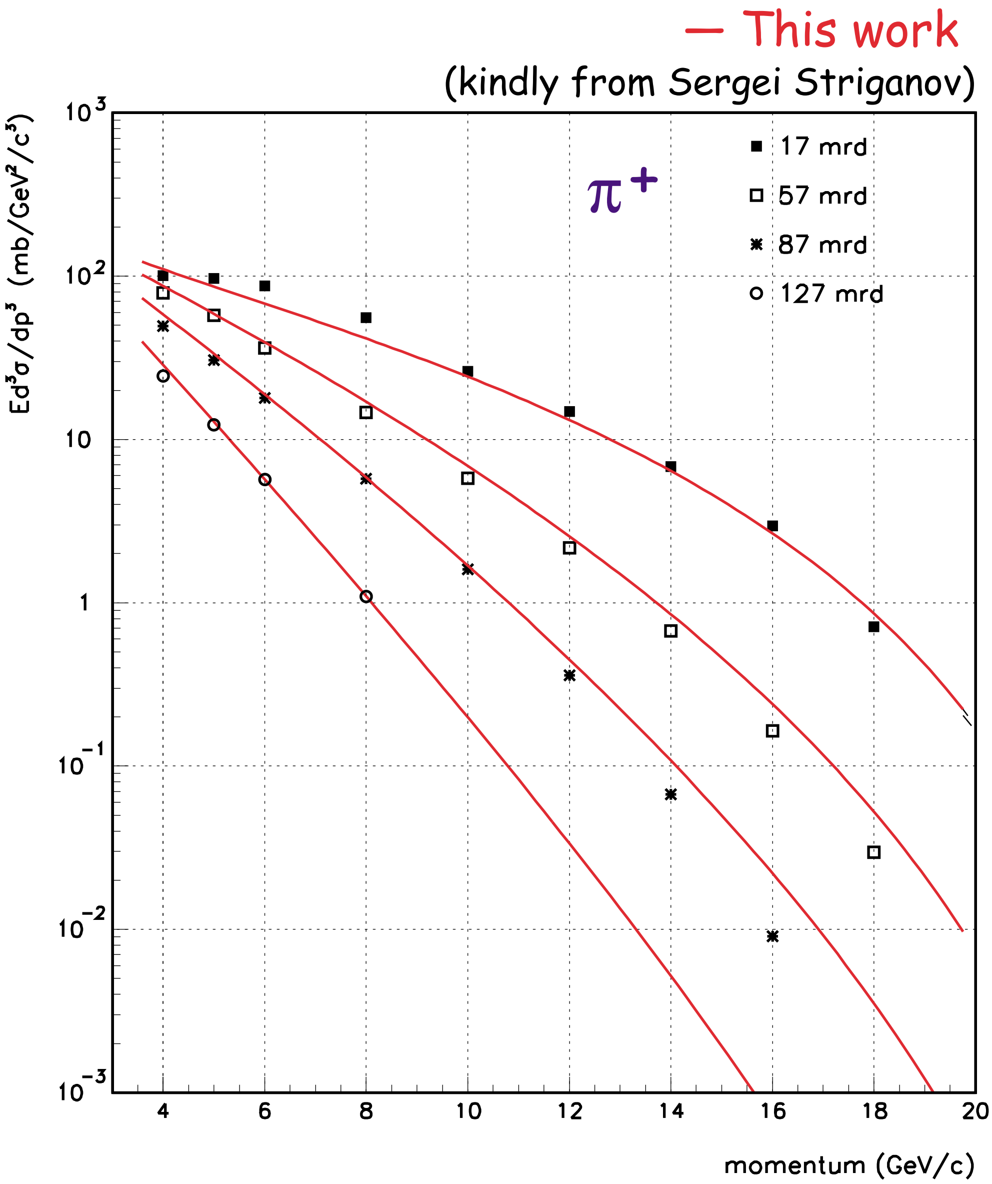
    (*D.S. Barton et al., Phys. Rev. D35 (1987) 35*)
  - Neglecting p_t dependence of α: $\leq$ 10% systematic error from Be to C
- Extension to different incident proton energy:
  - Comparison of the parameterization with available data taken protons on light material target:
    - 100 GeV/c proton on Carbon (*D.S. Barton et al., Phys. Rev. D27 (1983) 2580*)
    - 24 GeV/c proton on Beryllium (*T.Eichten et al., Nucl. Phys. B44 (1972) 333*)

Barton et al., 100 GeV/c protons on Carbon

— This work



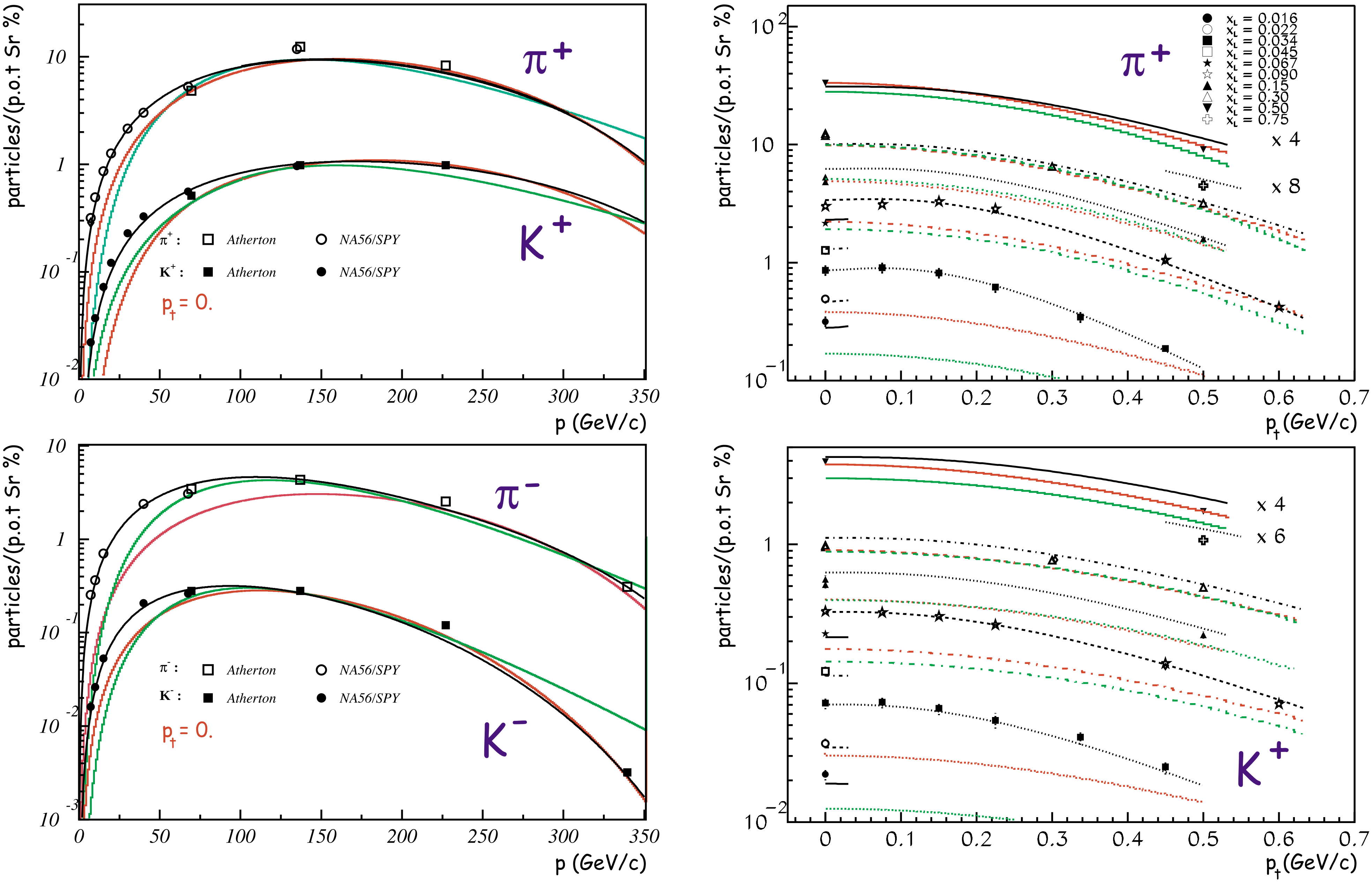
Eichten et al., 24 GeV/c protons on Beryllium



Secondary yield from 100 mm Beryllium target scaled at 450 GeV/c.

Comparison with existing parametrisations:

— This work;
— Atherton et al.;
— Malensek.



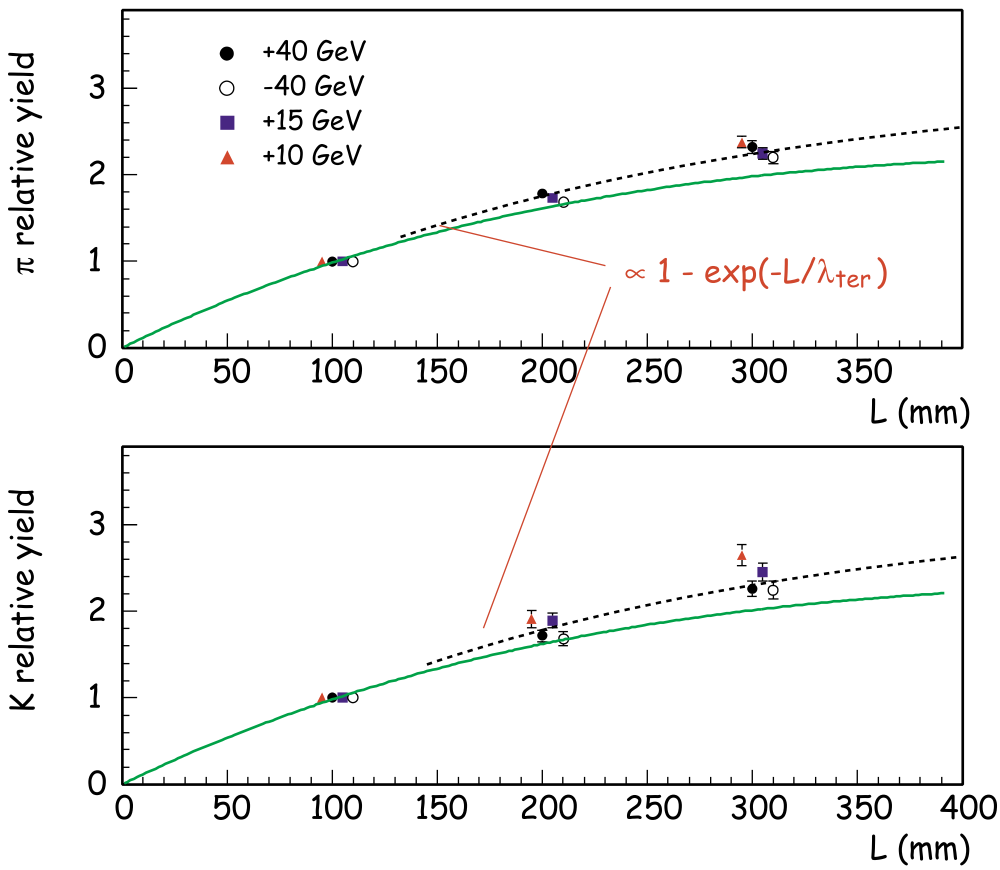
Evaluation of tertiary particles production on thick targets (SPY data)

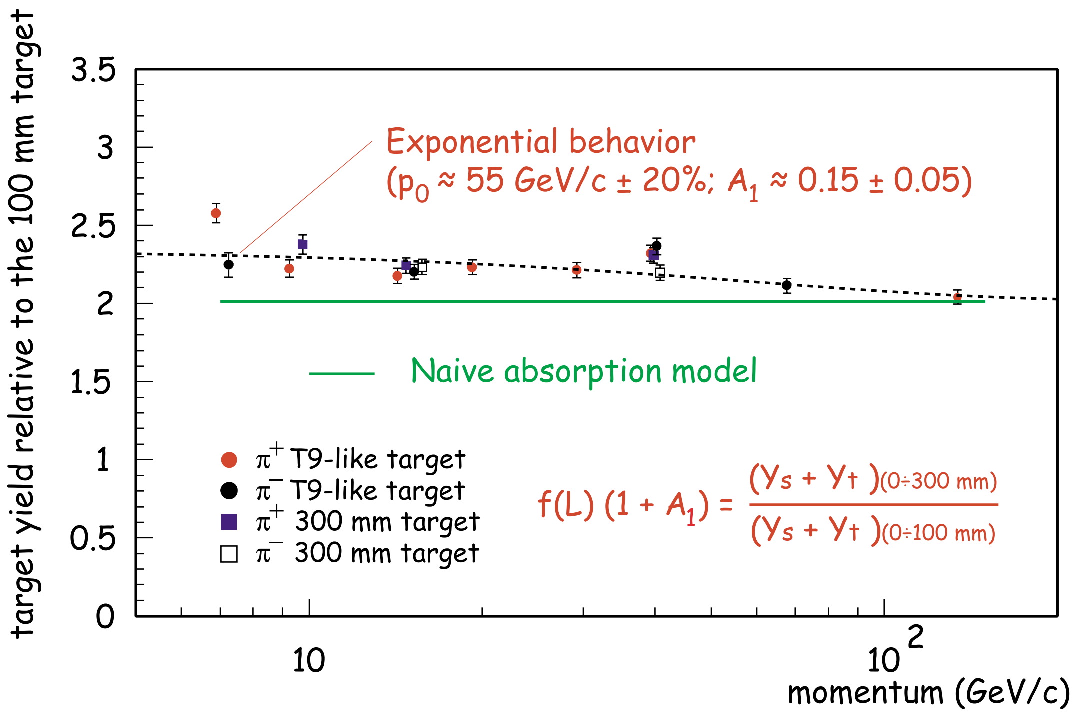
- Experimental evidences:
  - Comparison of secondary production from targets of different thickness (100, 200, 300 mm and T9) in SPY.
    - Naïve absorption model inadequate at large thickness
- Empirical assumptions (for long "needle-like" targets):
  - Tertiary particles produced mostly by re-interaction of leading secondary particle in the forward direction:
    - ≈ same p_t distribution as secondary.

$$N^*_{sec}(L,x,p_t) = N_{sec}(L,x,p_t)\cdot(1 + A_{ter}(1 - e^{-L/\lambda_{ter}})e^{-x/\xi})$$

- $\xi \approx 0.12 \pm 20\%$; from fit of data excess ($\pi^\pm$, $K^\pm$) at $p_t=0$ in the 300 mm & T9 targets w.r.t. the 100 mm.
- $A_{ter}(r_{target}) \approx 1.5 \pm 0.5$; from fit of data (300 mm / 100 mm).
- $\lambda_{ter}(E_{leading}, \langle p_t \rangle_{leading}, r_{target}) \approx 700$ mm; from integration of secondary particle yield at $E_{leading} \approx 0.4\,E_p$ over target radius.

Evaluation of tertiary production from yield of thick targets in SPY data



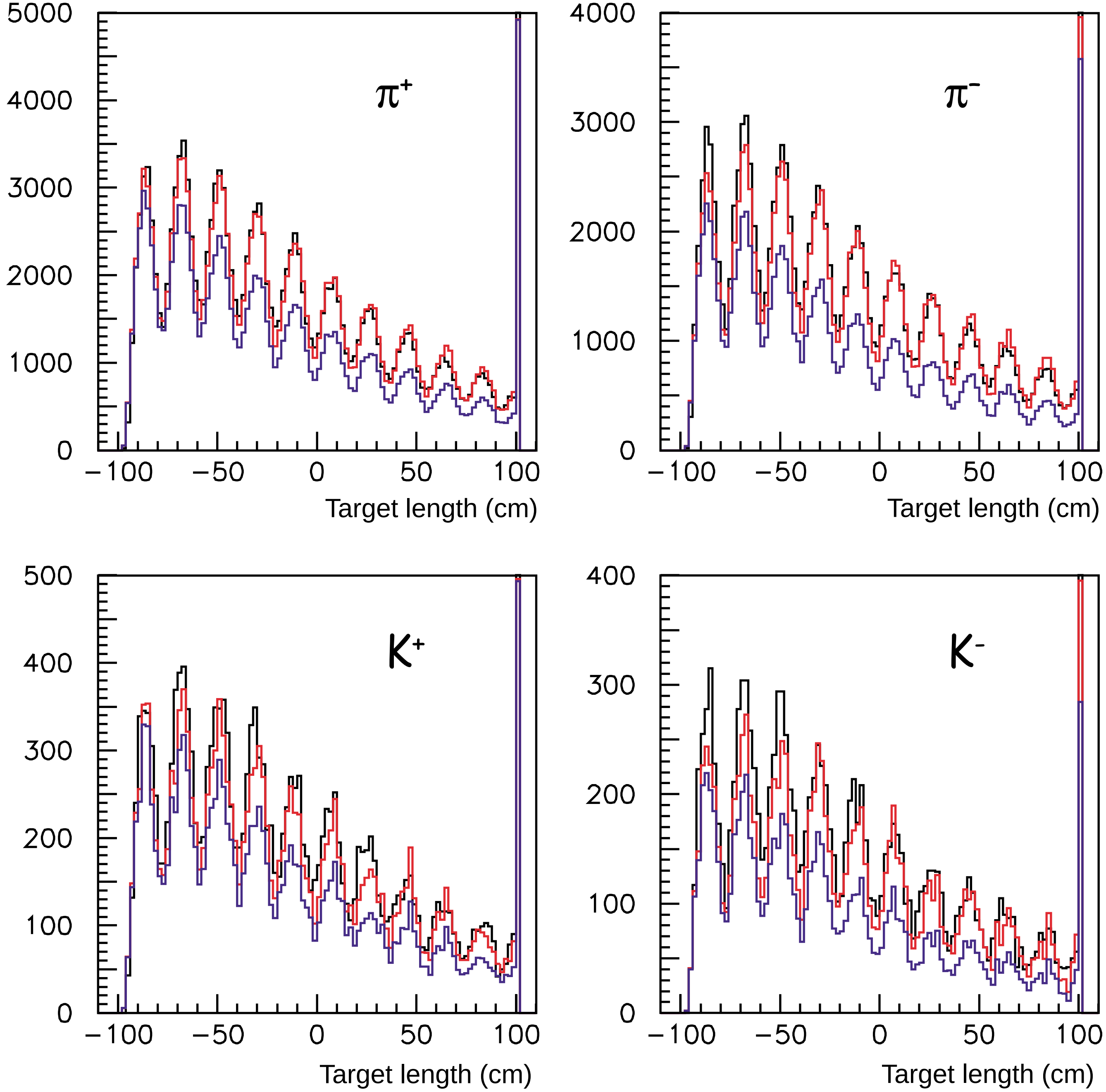


Secondary particle yield from WANF and CNGS targets

- Simple description of target layout & proton beam:
  - WANF Beryllium target: 11x(10cm-rods + 9cm-air); 3 mm diameter
  - CNGS Graphite target: 8x(10cm-rods + 9cm-air) + 48cm-rod; 4 mm diameter
  - Proton beam with nominal energy (CNGS: 400 GeV, WANF: 450 GeV), width and divergence.
- Proton interactions in target:
  - Secondary production ($\pi^{\pm}$, $K^{\pm}$ and K^0_L) from parameterization.
    - Simple exponential distribution along target bars.
    - Attenuation due to re-interactions along path length in target included.
  - Tertiary contribution from empirical model.
    - Continous target approximation:
      - $A_{ter}\,(1-\exp(-L/\lambda_{ter}))\,\exp(-x/\xi)$;
      - $\xi \approx 0.12$; A_{ter} and $\lambda_{ter} \propto$ target diameter.

WANF: Proton interactions in target

- 450 GeV protons
- Beryllium target
- 30 mrad acceptance

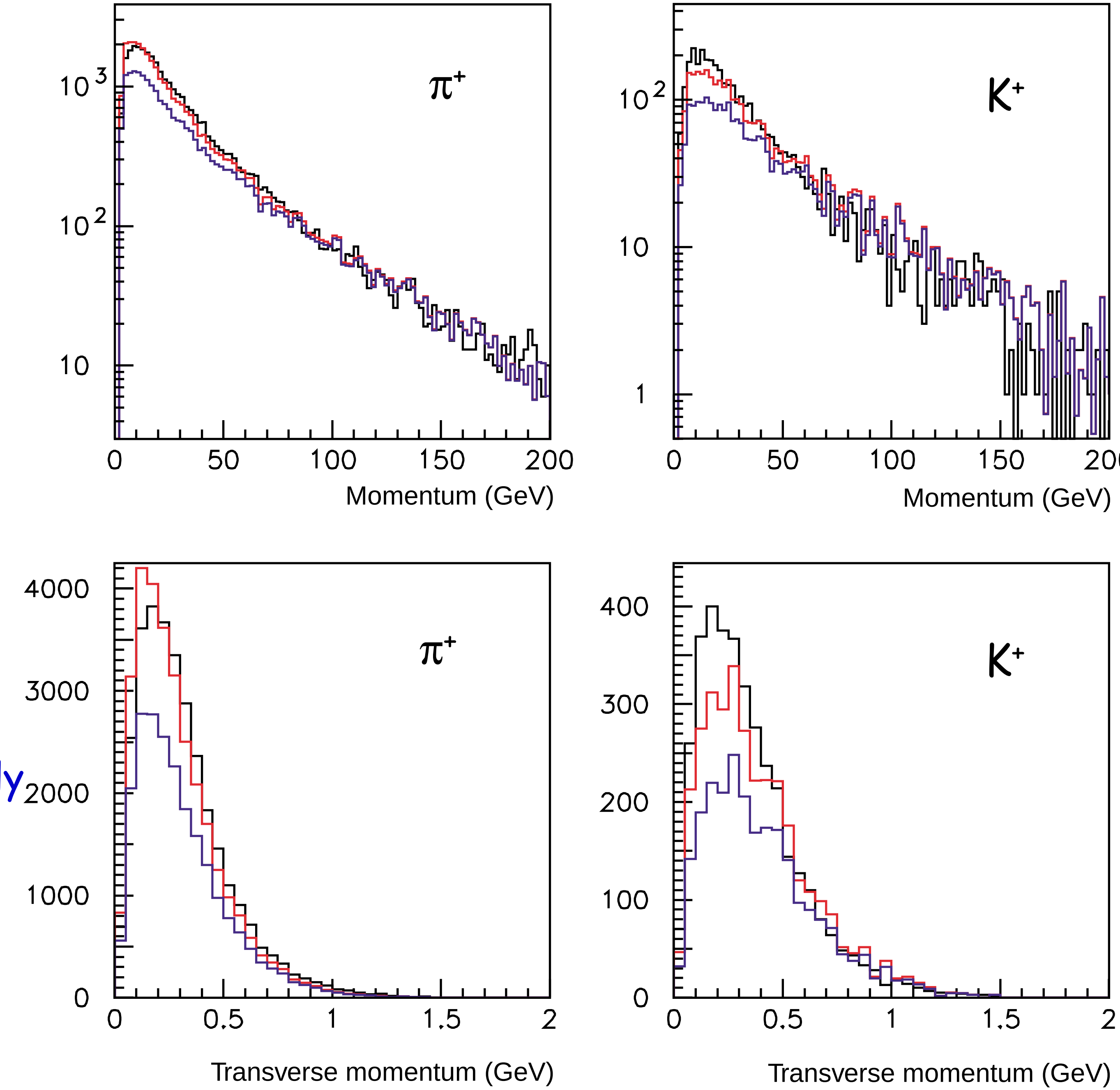


- Exit point from target:
  - Black: Fluka
  - Blue: Secondaries only
  - Red: with Tertiaries

WANF: Proton interactions in target

- 450 GeV protons
- Beryllium target
- 30 mrad acceptance

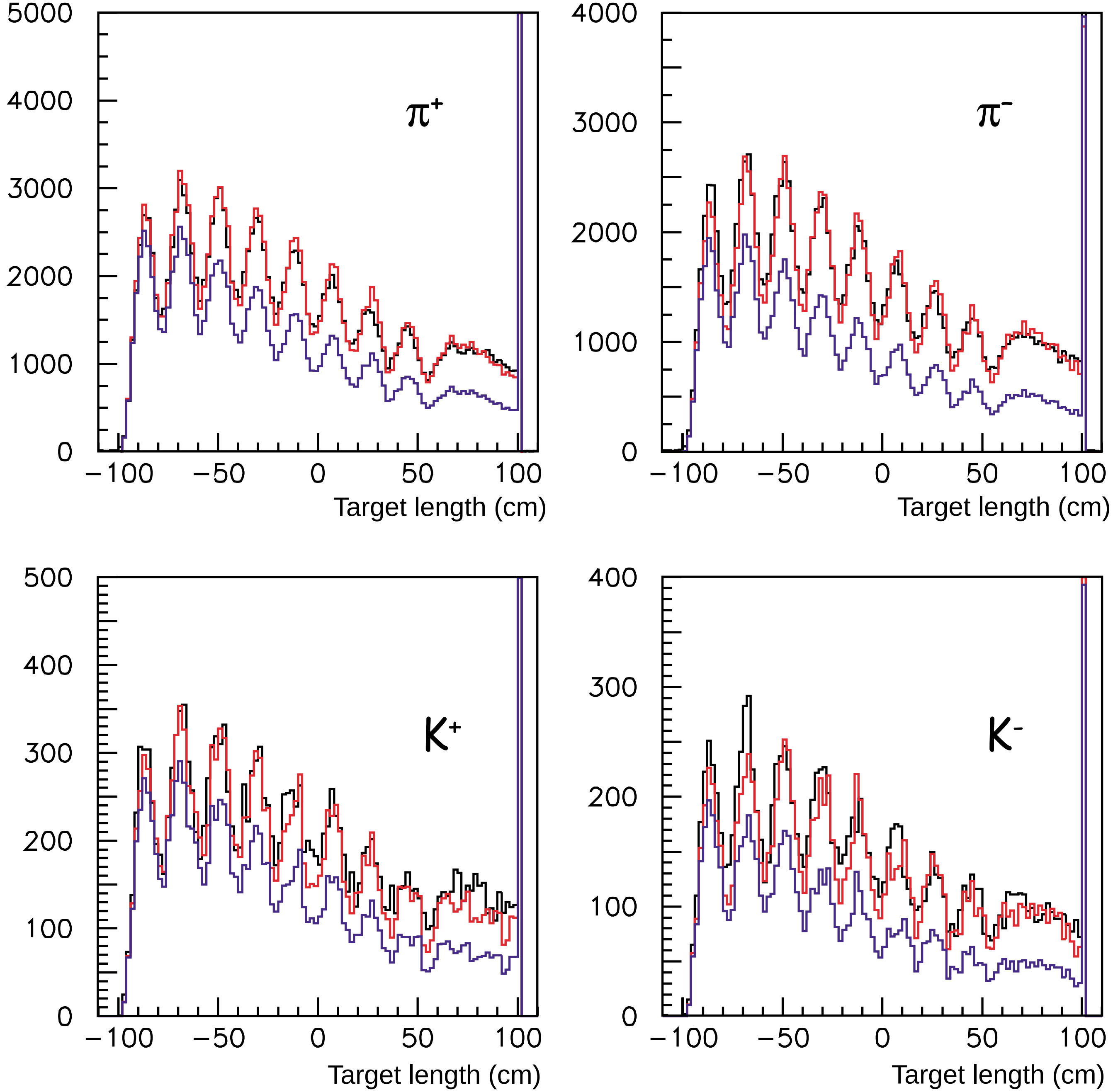
- p and p_T distributions:
  - Black: Fluka
  - Blue: Secondaries only
  - Red: with Tertiaries



CNGS: Proton interactions in target

- 400 GeV protons
- Graphite target
- 30 mrad acceptance

- Exit point from target:
  - Black: Fluka
  - Blue: Secondaries only
  - Red: with Tertiaries



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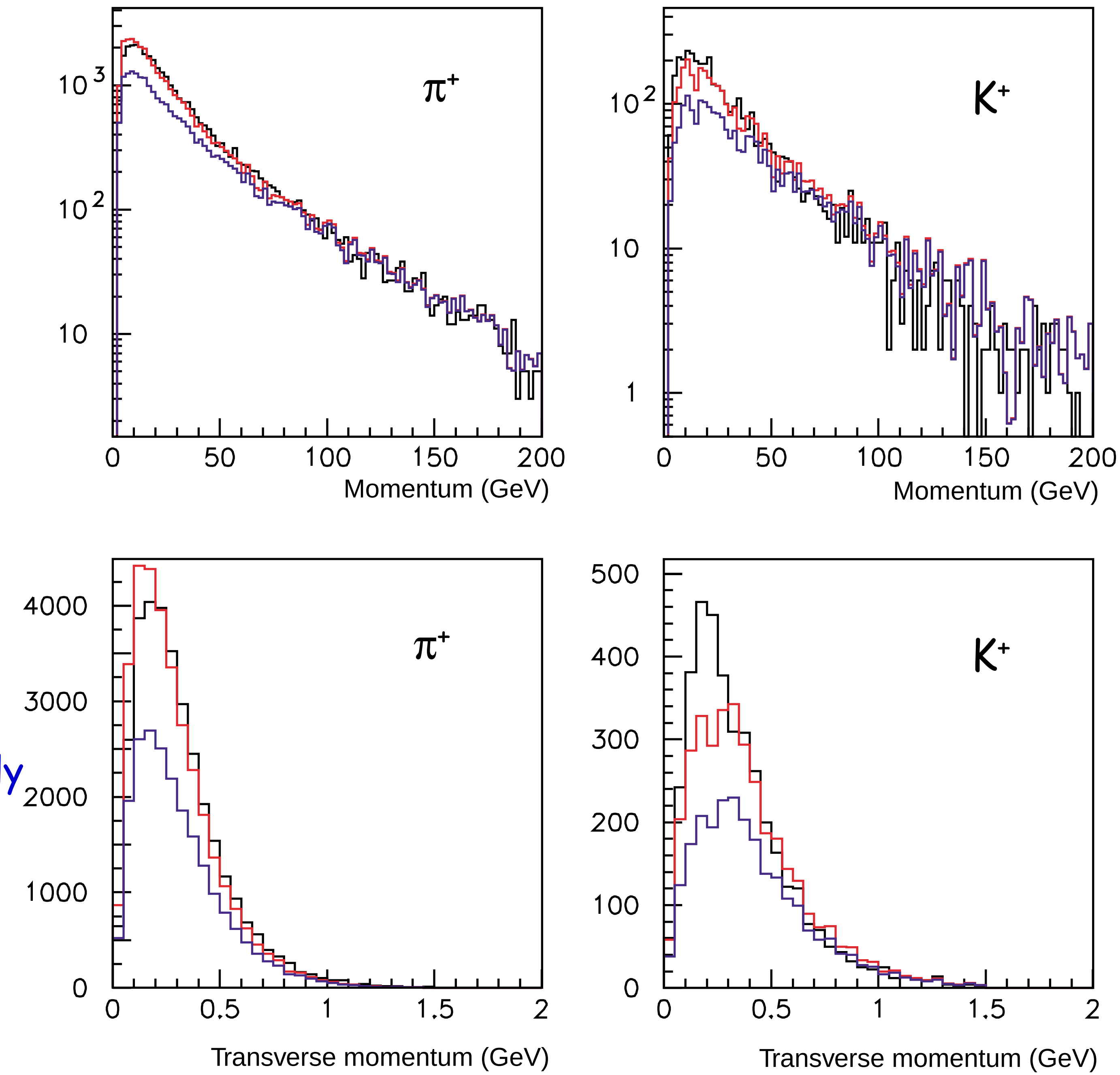


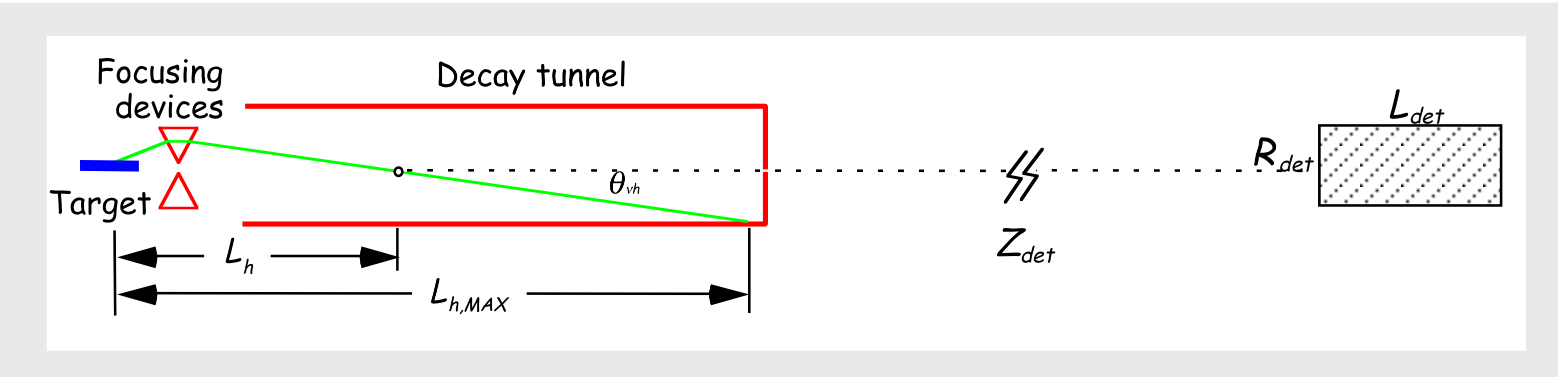
A fast neutrino beam simulation for any detector distance (with biased statistics)

- Simplified geometry of the beam-line
  - Tri-dimensional description of all beam line elements.
- Tracking of parent mesons:
  - Trajectory calculated up-to crossing with beam-line boundary.
    - Fine step tracking in magnetic field of Horn and Reflector.
  - Accurate evaluation of traversed material and multiple scattering.
    - Re-interaction and tertiary production in material along beam-line considered.
- Neutrino production:
  - Parent mesons forced to decay along trajectory.
    - 2/3 body decay included with correct kinematics and Branching Ratio.
  - Neutrinos forced to be emitted on detector direction.
    - Muon production and decay also considered.

A fast neutrino beam simulation: phase-space weighting

$$P = \left[\left(1 - e^{-\frac{L_{h,MAX}}{\lambda_h}}\right) e^{-\frac{L_h}{\lambda_h}}\right]\left[e^{-\frac{L_{mat}}{\lambda_{int}}}\right]\left[BR \rightarrow \begin{array}{c}\nu_\mu \mu \\ \nu_e X\end{array}\right]\left[\left(\frac{m_h}{E_h - p_h \cos\theta_{\nu h}}\right)^2\right]\left[\sigma_0 \frac{m_h^2 - m_{\mu,x}^2}{2m_h} \frac{m_h}{E_h - p_h \cos\theta_{\nu h}}\right]\left[\frac{\pi R_{det}^2}{4\pi Z_{det}^2} N_A \rho_{det} L_{det}\right]$$

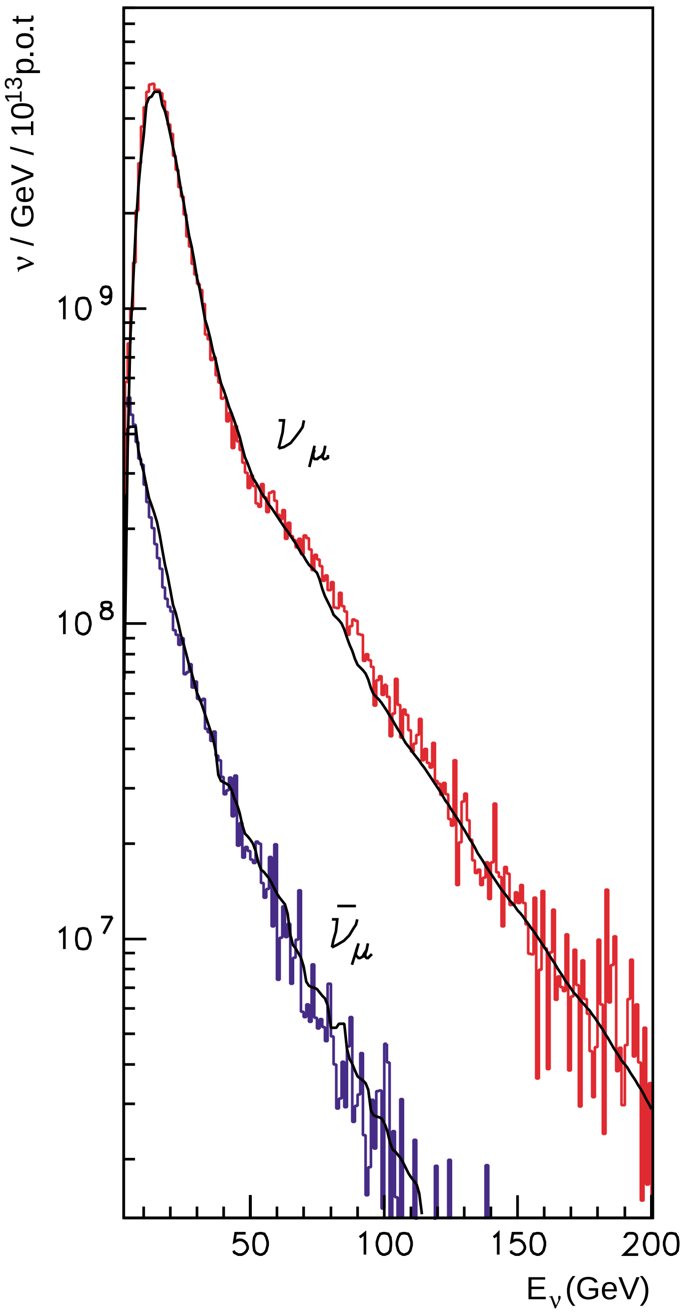
- **Hadron decay probability inside the tunnel**
- **Interact. in material**
- **2/3 body decay branching ratio**
- **Probability that the ν is emitted in the detector direction**
- **Total ν cross-section**
- **ν momentum in lab frame**
- **Solid angle**
- **Nucleon target density in the detector**

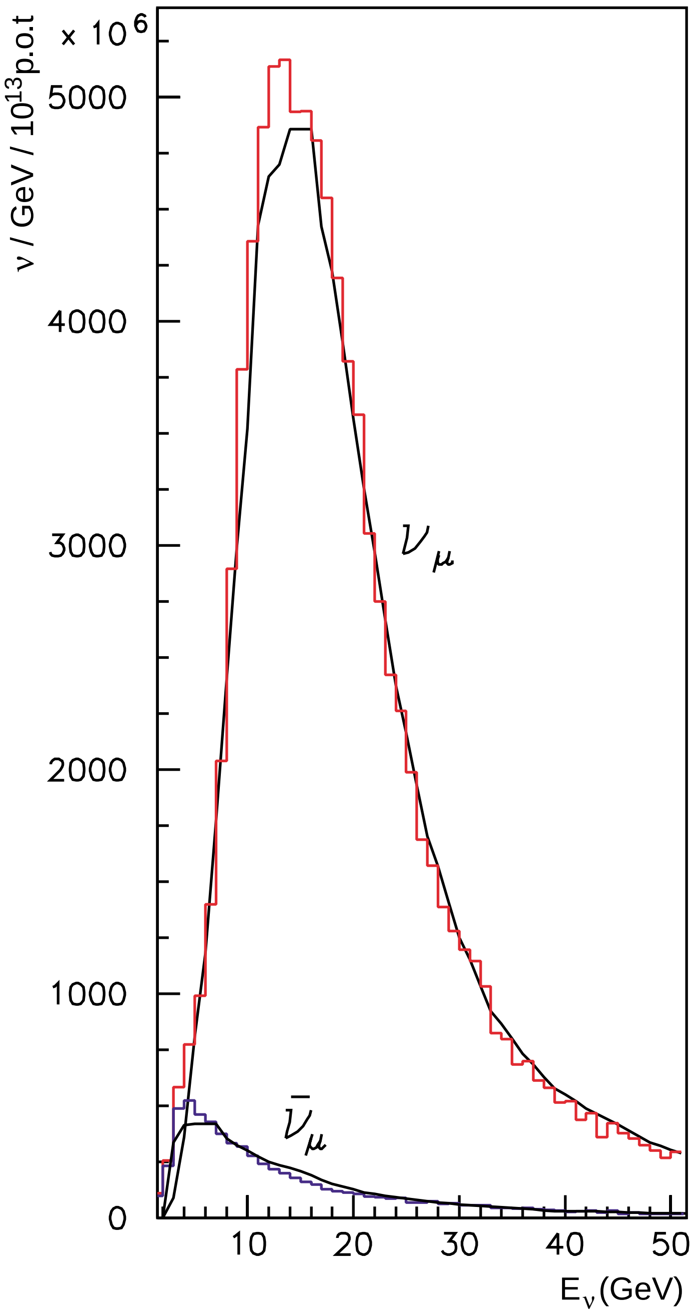


WANF: Comparison with CHARM II data

The CHARM II collab., Eur. Phys. J. C11 (1999), 18

- 450 GeV protons
- Be target
- 8 mrad acceptance
- Detector at 881 m
- Positive focusing
  - CHARM II data
  - ν_μ beam (10^6 pot)
  - Anti- ν_μ contamination





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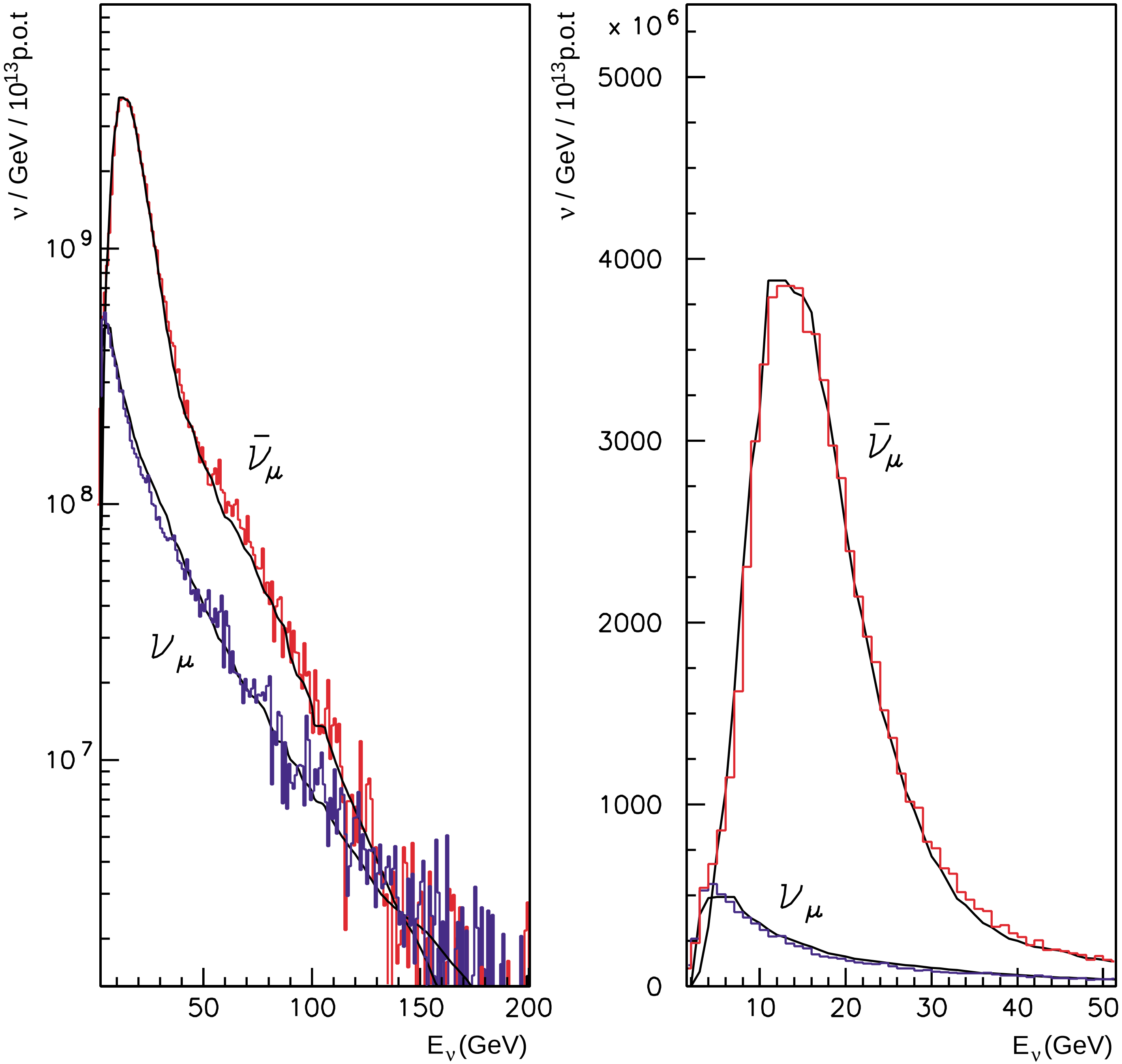
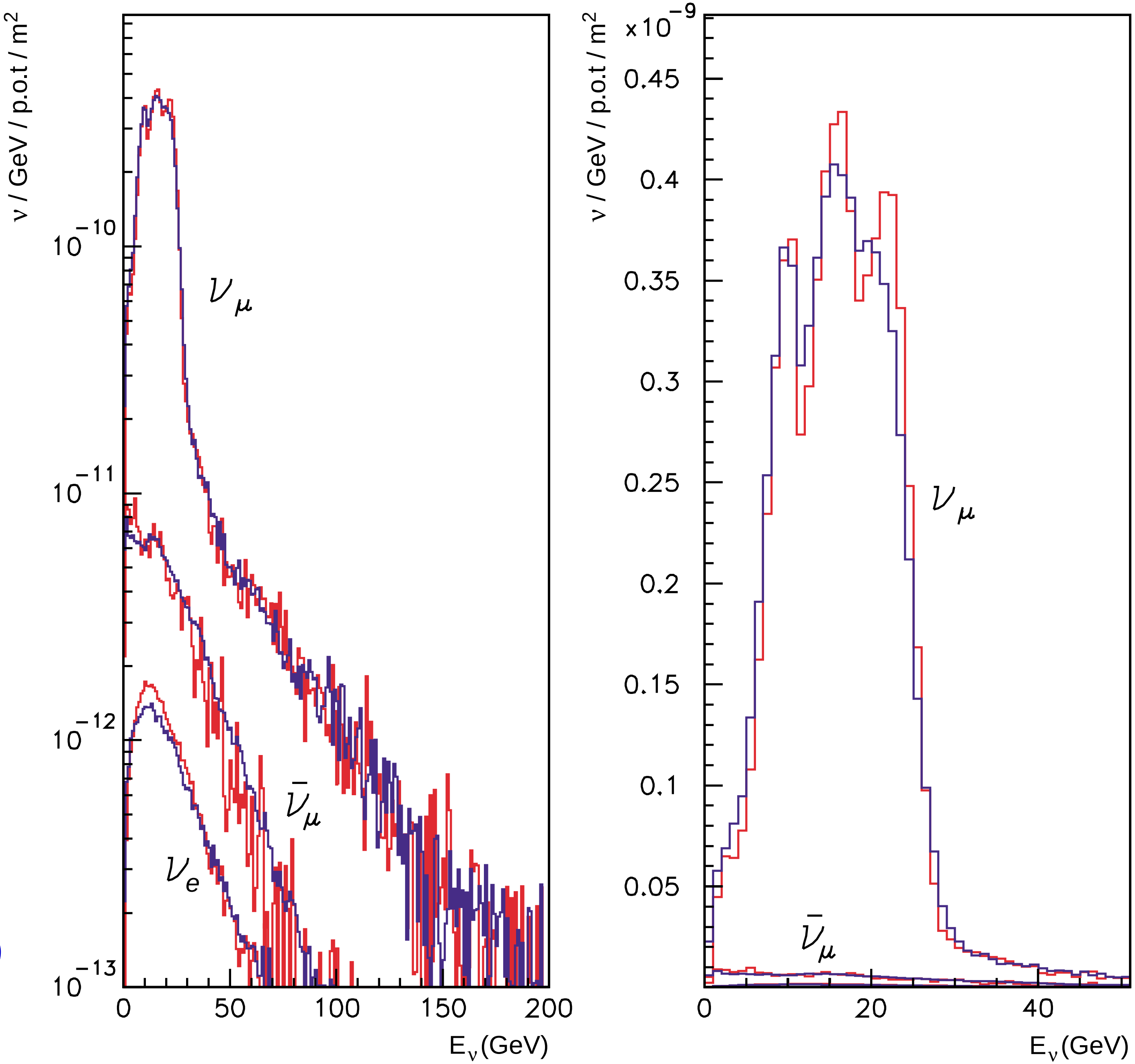


CNGS: Comparison with Fluka+Geant Monte Carlo

R. Bailey et al., CERN-SL-99-034-DI, 1999

- 400 GeV protons
- Graphite target
- Positive focusing
- Detector at 732 Km
  - This simulation
    - 10^5 pot
    - 10^2 m^2
  - Fluka+Geant MC
    - 10^6 pot
    - $\pi \cdot 100^2$ m^2 (ν_μ)
    - $\pi \cdot 400^2$ m^2 (anti-ν_μ)



Conclusions

- The proposed parameterization of secondary particles production on light nuclei:
  - Fits satisfactory the 400 & 450 GeV *p-Be* experimental data over a wide x and p_T range.
  - Can be safely extrapolated to thick target and to different proton momentum ($\rightarrow \approx$ 24 GeV/c) and target material
- Applied to neutrino beam simulations:
  - Efficient (fast) alternative to full hadronic cascade codes
  - Reproduces existing data within ≈ 10% (CHARM II and ...)
  - Validates full MC simulations of future beams (CNGS: FLUKA stand alone, FLUKA+GEANT)